

Spark Learning with Differentiation for Middle School Advanced Mathematics

Grade 8
Advanced Math
Handout packet

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Identifying Functions
Independent Practice

Part 1 Directions: Cut out the relationship cards. Sort the cards into two categories: those whose input values generate only one output value and those who do not. Use your categories to answer the debriefing questions.

Only One Output Value for Each x-value	More Than One Output Value for Some x-values
<p style="text-align: center;">Relationship Card</p>	<p style="text-align: center;">Relationship Card</p>
<p style="text-align: center;">Relationship Card</p>	<p style="text-align: center;">Relationship Card</p>



Relationship Card	Relationship Card
Relationship Card	Relationship Card
Relationship Card	Relationship Card



Debriefing Questions

ELPS Strategy: Discuss your ideas with a partner.

1. In a set of ordered pairs, how can you tell if each input value generates only one output value?
2. In a table, how can you tell if each input value generates only one output value?
3. In a mapping diagram, how can you tell if each input value generates only one output value?
4. In a graph, how can you tell if each input value generates only one output value?
5. If two different x -values generate the same y -value, does the relation meet the criteria that each x -value generates only one y -value? Explain your thinking.

Part 2 Directions: Some of the following relations are functions and some are not. Determine which ones are functions. For those that are not, explain how you know it is not a function and change the representation so that it is a function.

1.

x	y
1	4
5	16
10	31
12	37
15	46

Function? _____

2.

x	y
0	5
2	7
3	10
2	9
4	13

Function? _____

3.

x	y
1	6
0	5
2	6
3	8
4	9

Function? _____

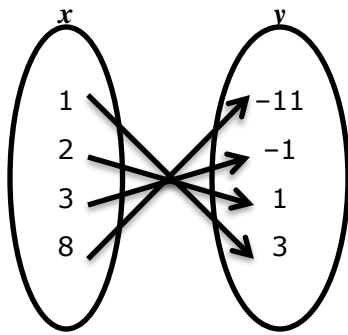
4. $\{(1, 5), (4, 11), (8, 10), (4, 5)\}$ Function? _____

5. $\{(1, 5), (12, 5), (20, 5), (31, 5)\}$ Function? _____

6. $\{(1, 5), (1, 6), (1, 8), (1, 10)\}$ Function? _____

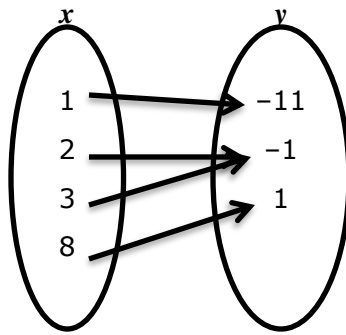


7.



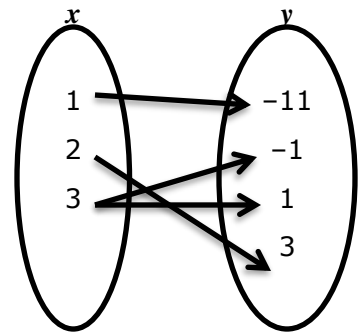
Function? _____

8.



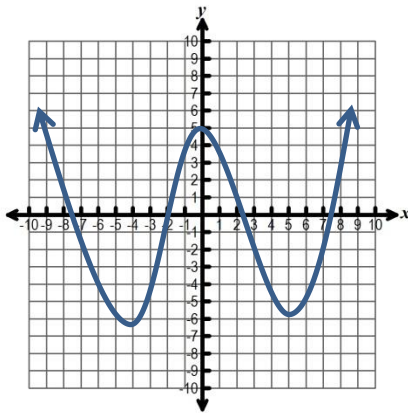
Function? _____

9.



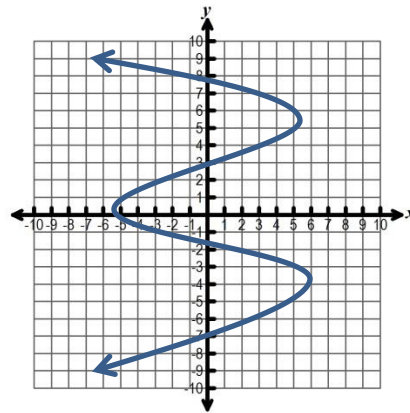
Function? _____

10.



Function? _____

11.



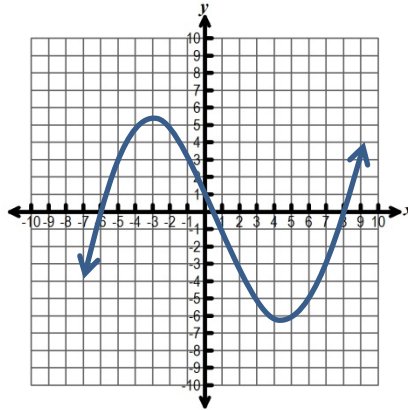
Function? _____



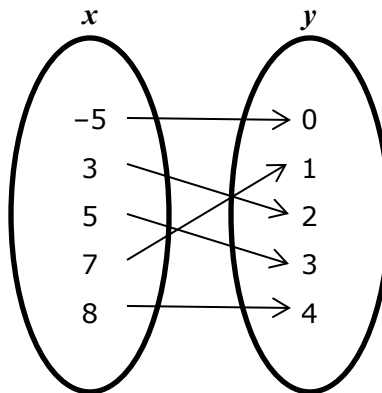
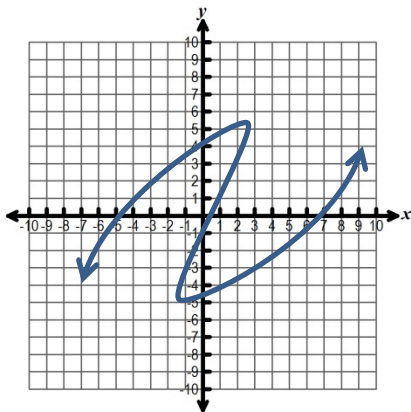
Identifying Functions

Independent Practice – Activity Master

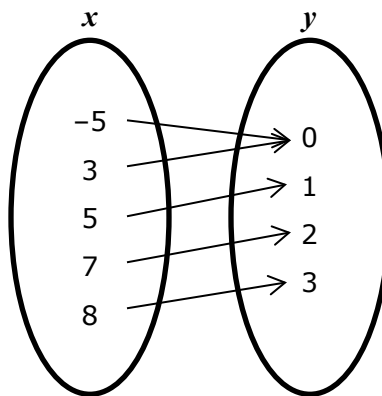
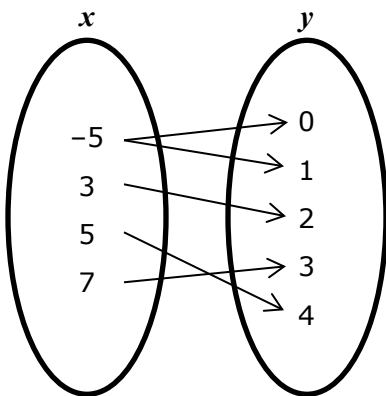
x	y
4	11
6	10
8	9
10	8
8	9



$\{(1, 5), (1, 4), (5, 8), (9, -4)\}$



x	y
4	11
6	10
8	9
10	8
8	8



$\{(1, 5), (3, 4), (5, 5), (9, 4)\}$



IDENTIFYING FUNCTIONS



The student is expected to identify functions using sets of ordered pairs, and mappings.



TELL ME MORE...

A **function** is a relationship between two variables where each input value (value of the independent variable) generates only one output value (value of the dependent variable). If the same input value in a relation generates different output values, then the relation is not a function.

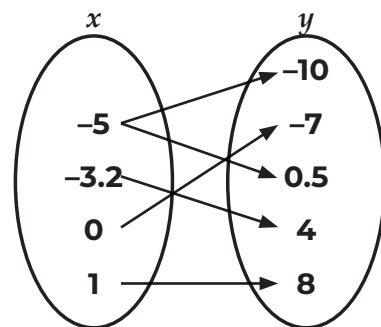
Functions	Not Functions																				
<p>$\{(2, 1), (3, 2), (5, 10), (8, 2)\}$</p> <p><i>Rationale:</i> Each input value (2, 3, 5, and 8) generates only one output value. Two input values, 3 and 8, both generate the same output value but 3 and 8 will each consistently generate an output value of 2.</p>	<p>$\{(2, 1), (3, 2), (5, 10), (8, 2), (3, 7)\}$</p> <p><i>Rationale:</i> The same input value, 3, generates two different output values, 2 and 7.</p>																				
<table border="1" style="width: 100%; text-align: center;"> <tr> <td style="background-color: #D3D3D3;">x</td> <td>1.5</td> <td>2</td> <td>3</td> <td>1.5</td> </tr> <tr> <td style="background-color: #D3D3D3;">y</td> <td>7</td> <td>10</td> <td>12</td> <td>7</td> </tr> </table> <p><i>Rationale:</i> Each input value generates only one output value. The input value 1.5 always generates an output value of 7.</p>	x	1.5	2	3	1.5	y	7	10	12	7	<table border="1" style="width: 100%; text-align: center;"> <tr> <td style="background-color: #D3D3D3;">x</td> <td>4.5</td> <td>7</td> <td>4.5</td> <td>10</td> </tr> <tr> <td style="background-color: #D3D3D3;">y</td> <td>8</td> <td>7</td> <td>6</td> <td>5</td> </tr> </table> <p><i>Rationale:</i> The same input value, 4.5, generates two different output values, 8 and 6.</p>	x	4.5	7	4.5	10	y	8	7	6	5
x	1.5	2	3	1.5																	
y	7	10	12	7																	
x	4.5	7	4.5	10																	
y	8	7	6	5																	
<div style="display: flex; align-items: center;"> <div style="flex: 1;"> </div> <div style="flex: 1; padding-left: 20px;"> <p><i>Rationale:</i> Each input value (x-value) generates only one output value (y-value).</p> </div> </div>	<div style="display: flex; align-items: center;"> <div style="flex: 1;"> </div> <div style="flex: 1; padding-left: 20px;"> <p><i>Rationale:</i> The same input value, 1.5, generates two different output values, 11 and 10.</p> </div> </div>																				
<div style="display: flex; align-items: center;"> <div style="flex: 1;"> </div> <div style="flex: 1; padding-left: 20px;"> <p><i>Rationale:</i> Each input value (x-value) generates only one output value (y-value).</p> </div> </div>	<div style="display: flex; align-items: center;"> <div style="flex: 1;"> </div> <div style="flex: 1; padding-left: 20px;"> <p><i>Rationale:</i> The same input value, 0, generates two different output values, 2 and -2.</p> </div> </div>																				

EXAMPLES

EXAMPLE 1: The mapping shows a relation between x and y . Is y a function of x ? Explain your answer.

STEP 1 Identify any input values, or x -values, that have more than one arrow from the mapping.

- When x is -5 , there are two arrows leading from -5 .
 - The other x -values all have one arrow.
- $x = -5$**



STEP 2 Determine whether or not the input value always has the same output value.

- The first arrow shows that $x = -5$ has an output value of $y = -10$.
- The second arrow shows that $x = -5$ has an output value of $y = 0.5$.

The output values for $x = -5$ are different.

STEP 3 Determine whether or not y is a function of x .

y is not a function of x because one input value, $x = -5$, has two different output values, $y = -10$ and $y = 0.5$. For y to be a function of x , each input value must generate only one output value and this is not true for $x = -5$.

EXAMPLE 2: Which of the sets of ordered pairs shown (if either) represents y as a function of x ?

Set I: $\{(0, 5), (1, 6), (2, 7), (3, 6)\}$

Set II: $\{(-1, 4), (0, 3), (1, 2), (0, 1)\}$

STEP 1 For Set I, identify any ordered pairs that have an x -value (input value) that is repeated.

- There are no ordered pairs with a repeated x -value.

Set I represents y as a function of x because each input (x) value generates only one output (y) value.

STEP 2 For Set II, identify any ordered pairs that have an x -value (input value) that is repeated.

- The ordered pairs $(0, 3)$ and $(0, 1)$ have the same x -value, 0 .

STEP 3 Determine whether the y -value (output value) for both ordered pairs is the same.

- For $(0, 3)$, when $x = 0$, $y = 3$.
- For $(0, 1)$, when $x = 0$, $y = 1$.

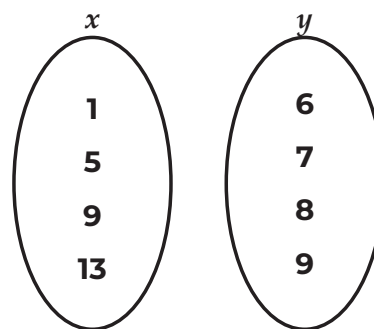
The y -values (output values) are different when $x = 0$.

YOU TRY IT!

Does the relation shown represent y as a function of x .

$\{(1, 6), (5, 7), (9, 8), (13, 9), (9, 7)\}$

Construct a mapping to represent the set of ordered pairs.



Identify any x -value that has more than one related y -value.

Is the relation a function? _____

STEP 4 Determine whether or not the set represents y as a function of x .

Set II does not represent y as a function of x because the input value of $x = 0$ generates two different output values, $y = 3$ and $y = 1$.

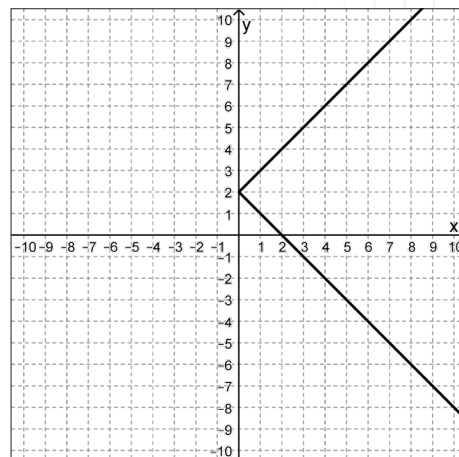
Only Set I represents y as a function of x .

EXAMPLE 3: Does the graph show a relation in which y is a function of x ? Explain.

STEP 1 On a graph, input values are x -values and output values are y -values. Determine any x -value that has more than one corresponding y -value.

- When $x = 0$, $y = 2$.
- For all other x -values, there are two y -values, one on each branch of the graph.

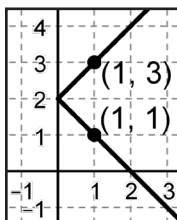
When $x > 0$ there are two y -values for each x -value



STEP 2 For $x = 1$, determine if the two y -values are the same.

- When $x = 1$, $y = 1$ or $y = 3$.

When $x = 1$, the y -values are not the same.



STEP 3 Determine if the graph represents y as a function of x .

The graph does not represent y as a function of x because when $x = 1$, there are two different y -values.

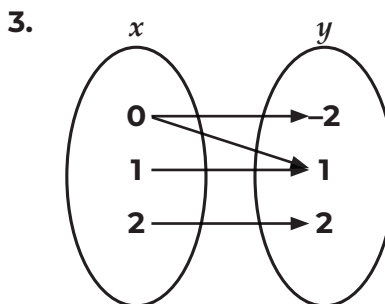


PRACTICE

For questions 1–8, determine if the relationship represents y as a function of x . If the relationship is not a function, explain why not.

1.

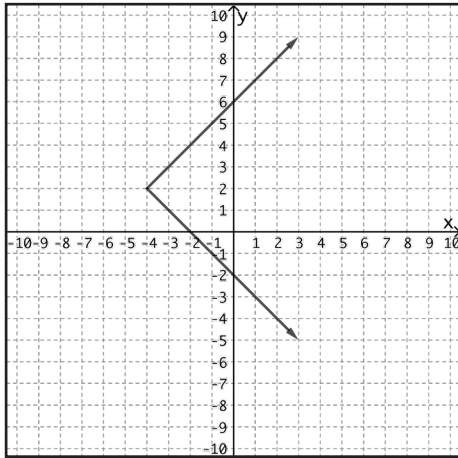
Input	-8	0	4	8
Output	4	6	7	8



2. $\{(-2, 4), (0, -2), (1, 1), (-2, -3)\}$

4. $\{(3, -1), (0, 0), (-3, 0), (-6, 2)\}$

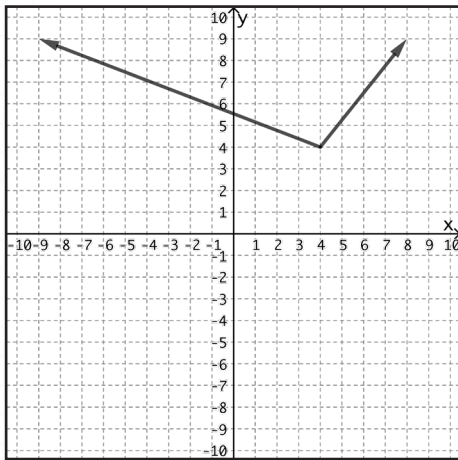
5.



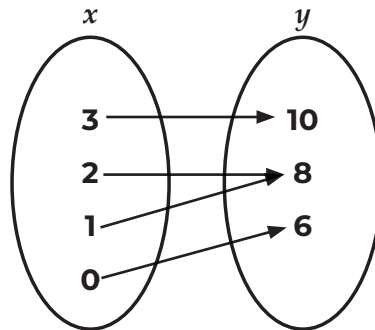
7.

x	y
1	-5
1	5
2	-10
3	15
4	-20

6.



8.



Use the relations below to answer questions 9–10.

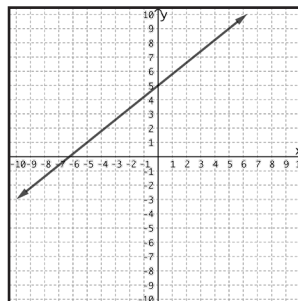
Relation A

x	y
-4	6
-1	6
0	6
2	6
5	6

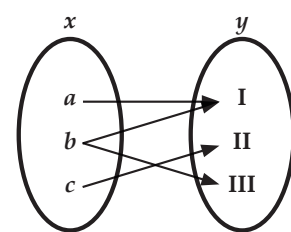
Relation B

$\{(-5, 4), (-1, 0), (3, -1), (3, 5)\}$

Relation C



Relation D



9. Which of the representations above shows a relation that is also considered a function?

10. Which of the representations above shows a relation that is NOT considered a function?

11. Which statement describes the table of values?

x	-3	-2	2	5	11
y	1	3	1	4	0

- A The table represents a function, because each y -value corresponds to only one x -value.
- B The table does not represent a function, because two of the x -values correspond to the same y -value.
- C The table represents a function, because each x -value corresponds to only one y -value.
- D The table does not represent a function, because two of the y -values correspond to the same x -value.

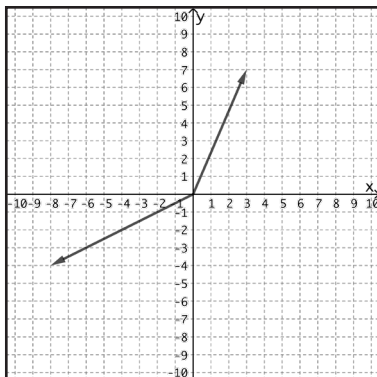
12. Which representation does NOT show y as a function of x ?

F $\{(-7, 1), (-3, 5), (1, 5), (7, -3)\}$

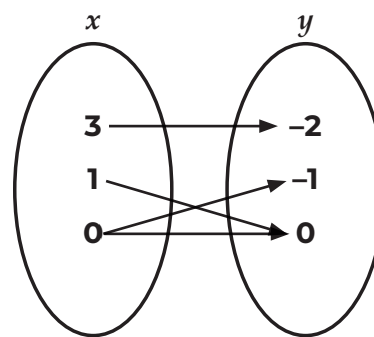
H

x	y
-2	3
-4	-2
3	3
5	-1

G



J



Performance Task: 8AM.12A
 Function Foundations: Area Codes and ZIP Codes

Telephone numbers in the United States all begin with an area code before the 7-digit phone number. Each area code is usually associated with a city or geographical area. Some large cities have more than one area code.

All mailing addresses in the United States include a 5-digit ZIP code at the end of the address. A ZIP code is a postal code used to group mail. The first digit represents a state or group of states. The first 3 numbers represent the central mail processing facility used to sort the mail in that state or geographic area. The last 2 digits correspond to a section or area of mail delivery such as specific post office area. Some large cities may have multiple ZIP codes.

Is the city to area code relationship a function or a relation? If it is not a function, how could the relationship be made a function? Is the relationship of city to ZIP code a function or a relation? If the relationship of a certain input to ZIP code is considered a function, what is the possible input? Justify your reasoning.

Procedural	0	1	2
Conceptual	0	1	2
Communication	0	1	2

Total points: _____



Performance Task: 8AM.12A
Function Foundations: Area Codes and ZIP Codes

Telephone numbers in the United States all begin with an area code before the 7-digit phone number. Each area code is usually associated with a city or geographical area. Some large cities have more than one area code.

Is the city to area code relationship a function or a relation? If it is not a function, how could the relationship be made a function? Justify your reasoning.

	0	1	2
Procedural	0	1	2
Conceptual	0	1	2
Communication	0	1	2

Total points: _____

Performance Task: 8AM.12A
 Function Foundations: Area Codes and ZIP Codes

Telephone numbers in the United States all begin with an area code before the 7-digit phone number. Each area code is usually associated with a city or geographical area. Some large cities have more than one area code.

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Is the city to area code relationship a function or a relation? If it is not a function, how could the relationship be made a function? Is the relationship of city to ZIP code a function or a relation? If the relationship of a certain input to ZIP code is considered a function, what is the possible input? How can the relationship between area code and telephone number be considered a function? Is it possible to make a function relationship between area code and ZIP code? Justify your reasoning.

Procedural	0	1	2
Conceptual	0	1	2
Communication	0	1	2

Total points: _____



Performance Task: 8AM.12A
Function Foundations: Area Codes and ZIP Codes

Telephone numbers in the United States all begin with an area code before the 7-digit phone number. Each area code is usually associated with a city or geographical area. Some large cities have more than one area code.

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1. What is the definition of a function?
2. How is a function different from a relation?
3. If a city name is the input and the area code is the output, is the city to area code relationship a function or a relation? Demonstrate using a table, mapping, sets of ordered pairs, and/or graph.



