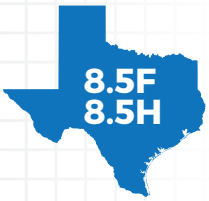
PROPORTIONAL AND NON-PROPORTIONAL RELATIONSHIPS



The student is expected to distinguish between proportional and non-proportional situations using tables, graphs, and equations in the form $y = kx$ or $y = mx + b$, where $b \neq 0$.

The student is expected to identify examples of proportional and non-proportional functions that arise from mathematical and real-world problems.

TELL ME MORE...

A **proportional relationship** is where paired values from the independent variable and dependent variable have a constant ratio. A linear relationship that is non-proportional is where the paired values do not have a constant ratio. In either case, both proportional relationships and non-proportional linear relationships have a constant rate of change of the dependent variable with respect to the independent variable.

PROPORTIONAL LINEAR RELATIONSHIPS

General form: $y = kx$

- k is the constant of proportionality
- k is equivalent to the constant rate of change

Attributes

- **Graph** contains the origin
- **Table** shows a constant ratio of paired values
- **Equation** does not have a constant term
- When $x = 0$, $y = 0$

NON-PROPORTIONAL LINEAR RELATIONSHIPS

General form: $y = mx + b$, where $b \neq 0$

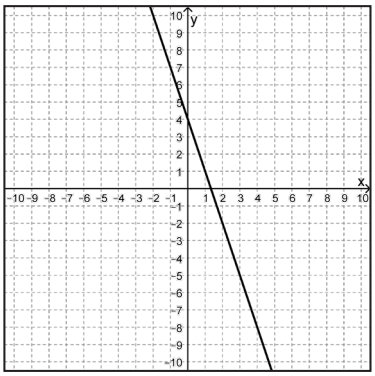
- m is the slope or constant rate of change
- b is the y-coordinate of the y-intercept

Attributes

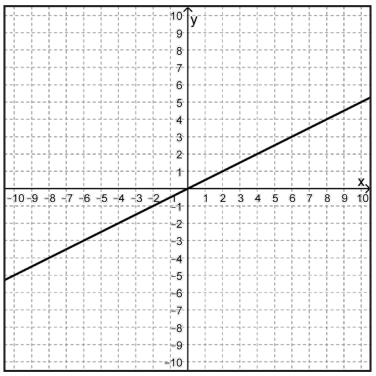
- **Graph** does not contain the origin
- **Table** shows a changing ratio of paired values
- **Equation** has a non-zero constant term
- When $x = 0$, $y = b$ ($b \neq 0$)

EXAMPLES

EXAMPLE 1: Which of the two graphs shown describes a proportional relationship? Which shows a non-proportional linear relationship? Explain your answer.



Graph I



Graph II

STEP 1 Both graphs show a line with a constant rate of change, so both relationships are linear. The graph of a proportional relationship passes through the origin, (0, 0). Identify the graph that contains (i.e., passes through) the origin.

Graph II shows a proportional relationship because the line contains the origin.

STEP 2 The graph of a non-proportional linear relationship has a constant rate of change but does not contain the origin. Identify the graph that has a constant rate of change and does not contain (i.e., pass through) the origin.

Graph I shows a non-proportional linear relationship because the line has a constant rate of change and does not contain the origin.

EXAMPLE 2: Which of the tables shows a proportional relationship? Which shows a non-proportional linear relationship?

Table 1

x	y
1	1.75
2	2
3	2.25
4	2.5
5	2.75

Table 2

x	y
0	0
1	1.5
2	6
3	13.5
4	24

Table 3

x	y
1	0.75
2	1.5
3	2.25
4	3
5	3.75

STEP 1 Analyze Table 1. Determine if it has a constant rate of change or a constant ratio of paired values.

x	y	
1	1.75	
		$2 - 1.75 = 0.25$
2	2	
		$2.25 - 2 = 0.25$
3	2.25	
		$2.5 - 2.25 = 0.25$
4	2.5	
		$2.75 - 2.5 = 0.25$
5	2.75	

$2 - 1 = 1$

$3 - 2 = 1$

$4 - 3 = 1$

$5 - 4 = 1$

Table 1 has a constant rate of change, $\frac{0.25}{1} = 0.25$, so it is linear.

x	y	$\frac{y}{x}$
1	1.75	$\frac{1.75}{1} = 1.75$
2	2	$\frac{2}{2} = 1$
3	2.25	$\frac{2.25}{3} = 0.75$
4	2.5	$\frac{2.5}{4} = 0.625$
5	2.75	$\frac{2.75}{5} = 0.55$

YOU TRY IT!

Determine whether the table shows a proportional relationship, non-proportional linear relationship, or neither. Justify your answer.

x	y
2	79
4	83
6	87
8	91
10	95

Constant rate of change? _____

Constant ratio? _____

Relationship: _____

Table 1 does not have a constant ratio of paired values so it is not proportional.

Table 1 shows a non-proportional linear relationship.

STEP 2 Analyze Table 2. Determine if it has a constant rate of change or a constant ratio of paired values.

x	y	
0	0	
		$1 - 0 = 1$ and $1.5 - 0 = 1.5$
1	1.5	
		$2 - 1 = 1$ and $6 - 1.5 = 4.5$
2	6	
		$3 - 2 = 1$ and $13.5 - 6 = 7.5$
3	13.5	
		$4 - 3 = 1$ and $24 - 13.5 = 10.5$
4	24	

Table 2 does not have constant rate of change so it is not linear.

Table 2 shows neither a proportional nor a non-proportional linear relationship.

STEP 3 Analyze Table 3. Determine if it has a constant rate of change or a constant ratio of paired values.

x	y	
1	0.75	
		$2 - 1 = 1$ and $1.5 - 0.75 = 0.75$
2	1.5	
		$3 - 2 = 1$ and $2.25 - 1.5 = 0.75$
3	2.25	
		$4 - 3 = 1$ and $3 - 2.25 = 0.75$
4	3	
		$5 - 4 = 1$ and $3.75 - 3 = 0.75$
5	3.75	

Table 3 has a constant rate of change.

Table 3 shows a proportional relationship.

x	y	$\frac{y}{x}$
1	0.75	$\frac{0.75}{1} = 0.75$
2	1.5	$\frac{1.5}{2} = 0.75$
3	2.25	$\frac{2.25}{3} = 0.75$
4	3	$\frac{3}{4} = 0.75$
5	3.75	$\frac{3.75}{5} = 0.75$

Table 3 has a constant ratio of paired values so it is proportional.

EXAMPLE 3: Does the situation shown represent a proportional or non-proportional linear relationship? How do you know?

The cost of purchasing n gallons of gasoline is \$2.19 per gallon plus a fuel tax of \$0.35 per gallon.

STEP 1 Write an equation to represent the situation.

- The cost of 1 gallon of gasoline is \$2.19(1). The cost of 2 gallons of gasoline is \$2.19(2). The cost of n gallons of gasoline is \$2.19n.
- The fuel tax for 1 gallon of gasoline is \$0.35(1). The fuel tax for 2 gallons of gasoline is \$0.35(2). The fuel tax for n gallons of gasoline is \$0.35n.
- Let C represent the combined cost of the gasoline and fuel tax for n gallons of gasoline.
- $C = 2.19n + 0.35n$

C* = 2.54*n

STEP 2 Determine whether the equation is of the form $y = kx$ for a proportional relationship or $y = mx + b$ where $b \neq 0$ for a non-proportional linear relationship.

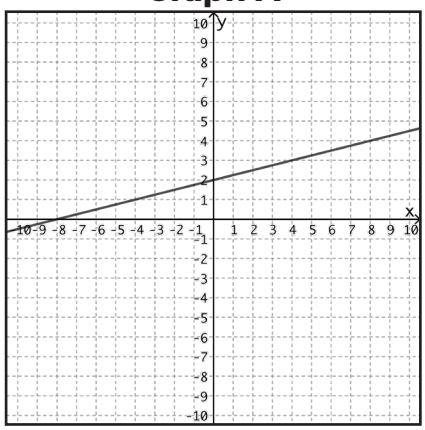
- The equation, $C = 2.54n$, has no constant term.
- The equation matches the form $y = kx$.

The situation represents a proportional relationship.

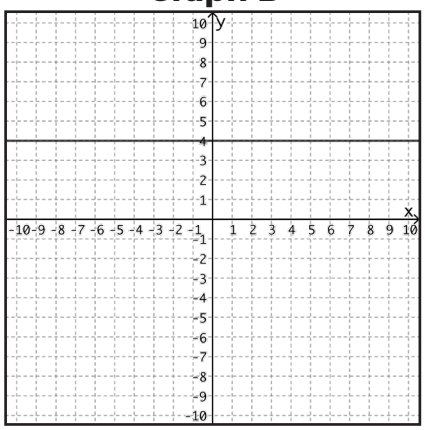
PRACTICE

Use the following graphs to answer questions 1–2.

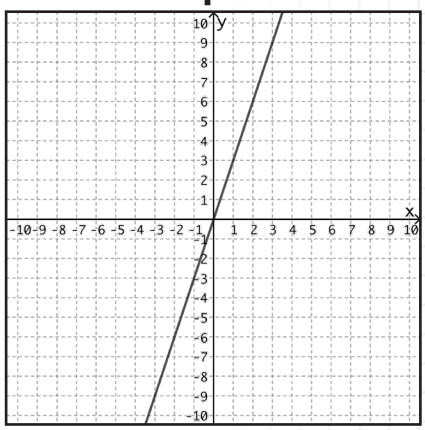
Graph A



Graph B



Graph C



1. Given the graphs shown above, which graph(s) show(s) a proportional relationship?

2. Given the graphs shown above, which graph(s) show(s) a non-proportional relationship?

Use the following tables to answer questions 3-4.

Table A

x	y
1	6
2	7
3	8
4	9

Table B

x	y
3	1
12	4
18	6
33	11

Table C

x	y
8	8
14	12.5
28	23
44	35

Table D

x	y
4	24
13	78
25	150
39	234

3. Given the tables shown above, which table(s) show(s) a proportional relationship?

4. Given the tables shown above, which table(s) show(s) a non-proportional relationship?

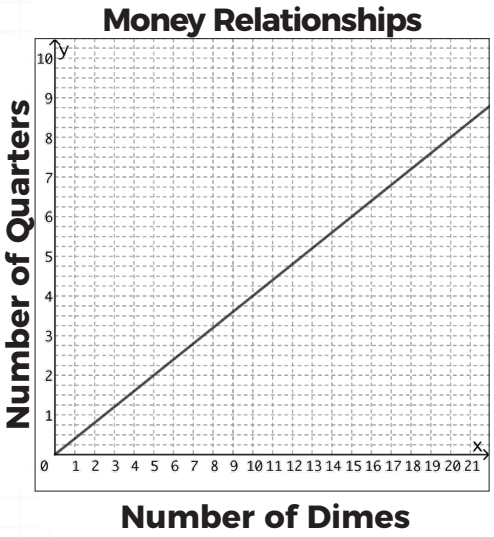
Use the following equations to answer questions 5-6.

Equation A	Equation B	Equation C	Equation D	Equation E
$y = -2$	$y = \frac{5}{2}x$	$y = \frac{2}{3}x + 4$	$y = 3x + 5$	$y = 4x$

5. Given the equations shown above, which equation(s) show(s) a proportional relationship?

6. Given the equations shown above, which equation(s) show(s) a non-proportional relationship?

7. The graph below shows the relationship between the number of dimes and the number of quarters that produce an equivalent amount of money. Is the relationship shown in the graph a proportional or non-proportional relationship? Explain how you know.



8. The table below shows costs for a person attending the local carnival, including the price of ride and food tickets. Based on the information in the table, is the relationship between cost and number of tickets purchased proportional or non-proportional? Explain how you know.

Tickets Purchased, x	Total Cost, y
10	$12.50
16	$20.00
28	$35.00
50	$62.50

9. Aaron uses the equation $y = 500 + 0.2x$ to determine his weekly paycheck based on his sales volume. Does Aaron's equation represent a proportional or a non-proportional relationship? Explain how you know.

10. Which of the following situations represents a proportional relationship?

A The amount Andrew earns mowing lawns for $18 per hour.

B The cost of baseball tickets online at $7 per ticket plus a $3 online processing fee.

C The total cost of a taxi that charges $2 per mile plus a passenger fee of $1.50 to pick up a rider.

D The height of a 10-inch candle after burning at a rate of $\frac{3}{4}$-inch each hour.

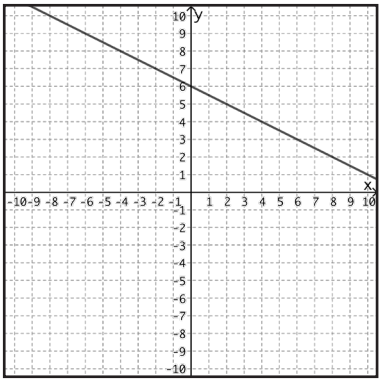
11. Which of the following representations shows a non-proportional relationship?

F

x	y
17	51
21	63
26	78
36	108

G $y = \frac{3}{8}x$

H



J The number of cans of sodas in x packages of 12 each.