

Cluster 8.5: Proportionality

8.5F: Proportional & Non-Proportional Relationships: Beach Rentals

Focusing TEKS

8.5F Proportionality. The student applies mathematical process standards to use proportional and non-proportional relationships to develop foundational concepts of functions. The student is expected to distinguish between proportional and non-proportional situations using tables, graphs, and equations in the form of $y = kx$ or $y = mx + b$, where $b \neq 0$. **Supporting Standard**

Additional TEKS:

8.5A Represent linear proportional situations with tables, graphs, and equations in the form of $y = kx$. **Supporting Standard**

8.5B Represent linear non-proportional situations with tables, graphs, and equations in the form of $y = mx + b$, where $b \neq 0$. **Supporting Standard**

8.9A Identify and verify the values of x and y that simultaneously satisfy two linear equations in the form of $y = mx + b$ from the intersections of the graphed functions. **Supporting Standard**

Focusing Mathematical Process

8.1A Apply mathematics to problems arising in everyday life, society, and the workplace.

8.1B Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution.

8.1D Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

8.1F Analyze mathematical relationships to connect and communicate mathematical ideas.

▲ Performance Task

The Jameson family of 5 and the Maritz family of 4 are spending the day at the beach. There they find a vendor renting some items they might need for the day. The Jameson's decide to get chairs for each person in their family and to share 3 umbrellas with each other for their day on the beach. The Maritz family also gets chairs for each person as well as 2 umbrellas to share. The Maritz family also rents the sandcastle tools for the day. Use at least two representations, to represent the cost of each family's rentals. Is each family's cost proportional to the number of hours they spend at the beach? At how many hours would the two families' costs be the same? Justify your reasoning.

Answer: The Jameson family cost can be represented as $y = \$25x$ as an equation, table, or graph (see student work). The Maritz family cost can be represented as $y = \$18x + 14$ as an equation, table, or graph (see student work). The Jameson family's cost equation is proportional while the Maritz family's equation is not proportional. The costs for the two family's rentals are the same at 2 hours.



Cluster 8.5: Proportionality

Mathematically Speaking...

In this task, students review a situation where two families are renting equipment at a beach. Students determine a representation for each family's rental costs based on the time spent at the beach. Students are expected to use multiple representations for each rental situation including equations, tables, and/or graphs. Once the representations are created, a comparison is made to determine the number of hours where the rental costs are equal for the two families. Finally, students review the representations for each situation and determine if each is proportional or non-proportional.



Students begin the foundation for proportions in grade 5 as they explore multiplicative relationships. Once students enter grade 6 mathematics, they work with multiplicative comparisons, rates, and proportions as a means to solve problems. Students continue this work throughout grade 7 when discovering the constant of proportionality, and with writing linear equations.

Possible Solution

For the Jameson family, there are 5 family members, and they each rented a chair. The chair cost is \$3 for 1 chair or \$5 for a pair (2) of chairs. The lowest cost for renting chairs would be if the family rented 2 pairs of chairs and 1 single chair for a total of \$13 ($5 + 5 + 3$) per hour.

The family also rented 3 umbrellas at a rate of \$4 per hour. The cost of all 3 umbrellas is \$12 per hour.

Adding together the cost per hour of the chairs and the umbrellas, the family will spend \$25 per hour for their rentals.

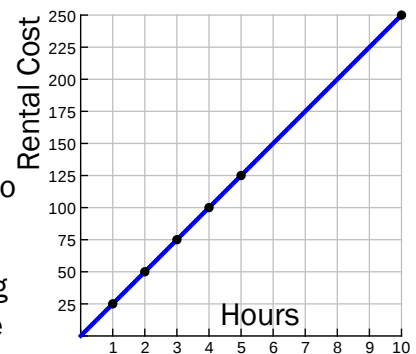
One way to represent the cost is to use a table where the input variable, h , represents the number of hours, and the output variable, c , represents the total cost. A process row helps to show the calculations for each cost based on the number of hours.

Hours spent at beach, h	1	2	3	4	5	10
Process	$1 \bullet 25$	$2 \bullet 25$	$3 \bullet 25$	$4 \bullet 25$	$5 \bullet 25$	$10 \bullet 25$
Rental cost, c	\$25	\$50	\$75	\$100	\$125	\$250

The equation to find the cost of the Jameson's rentals based on any number of hours is found using the process. In the table, each cost is the number of hours times the \$25 rental charge for the hour. The equation is $c = 25 \bullet h$ or $c = 25h$. In terms of x and y variables, the x variable is always the input, and the y is always the output. The equation using these variables is $y = 25x$.

Cluster 8.5: Proportionality

Another way to represent the cost for the Jameson family is to use a graph. Plot the equation $y = 25x$ using a coordinate plane. Start with plotting the points from the table. Then graph the equation using the points to help draw the line. The slope of the line is the unit rate of \$25 per hour for the rental charges. The y-intercept of the line is the origin, because if the Jameson's do not use the equipment, there is no charge.



The graph shows a straight line having a constant slope of 25 passing through the origin. This means the situation is proportional. When the situation is proportional, a proportion can be used to solve problems.

For example, each hour costs \$25 to rent the 5 chairs and 3 umbrellas. If the Jameson's spend 6 hours at the beach, a proportion can be set up and solved to find the rental cost for 6 hours.

$$\frac{1 \text{ hour}}{\$25} = \frac{6 \text{ hours}}{c}$$

Using equivalent fractions the cost of 6 hours is $25 \bullet 6$ or \$150.

For the Maritz family, there are 4 family members, and they each rented a chair. The chair cost is \$3 for 1 chair or \$5 for a pair (2) of chairs. The lowest cost for renting chairs would be if the family rents 2 pairs of chairs for a total rental cost of \$10 ($5 + 5$) per hour.

The family also rented 2 umbrellas at a rate of \$4 per hour. The total rental cost for 2 umbrellas is \$8 per hour.

Adding together the cost per hour of the chairs and the umbrellas, the family will spend \$18 per hour for their chair and umbrella rentals.

The Maritz family also decided to rent the sandcastle building tools. The rate for this is \$14 per day. This means that the Maritz family will pay \$14 for the sandcastle building tools no matter how many hours they spend at the beach on that day at a cost of \$14 for any number of hours spent at the beach that day.

One way to represent the cost is to use a table where the input variable, h , represents the number of hours and the output variable, c , represents the total cost. A process row helps to show the calculations for each cost based on the number of hours.

Hours spent at beach, h	1	2	3	4	5	10
Process	$1 \bullet 18 + 14$	$2 \bullet 18 + 14$	$3 \bullet 18 + 14$	$4 \bullet 18 + 14$	$5 \bullet 18 + 14$	$10 \bullet 18 + 14$
Rental cost, c	\$32	\$50	\$68	\$86	\$104	\$194

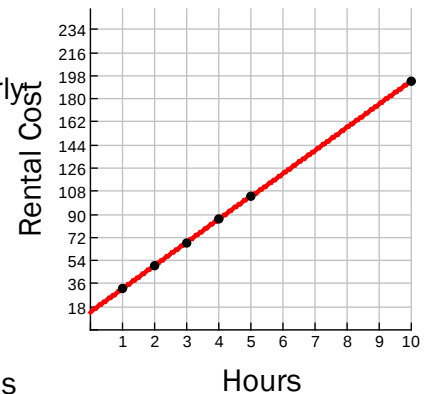
The equation to find the cost of the Maritz's rentals based on any number of hours is found using the process. In the table, each cost is the number of hours times the \$18 rental charge for the hours using the chairs and umbrellas **plus** the \$14 rental charge for the sandcastle tools.

The equation is $c = 18 \bullet h + 14$ or $c = 18h + 14$. In terms of x and y variables, the x variable is always the input, and the y is always the output. The equation using these variables is $y = 18x + 14$.

Cluster 8.5: Proportionality

Another way to represent the cost for the Maritz family is to use a graph. Plot the equation $y = 18x + 14$ using a coordinate plane. Start with plotting the points from the table. Then graph the equation using the points to help draw the line. The slope of the line is the hourly rate for the rental of the chairs and umbrellas at \$18 per hour. The y-intercept of the line is the point (0, 14). If the Maritz's do not use the equipment with the hourly charges, there is still an equipment charge of \$14 to use the sandcastle building tools for the day. This amount is added to the hourly rented equipment charge regardless of the number of hours spent at the beach.

The graph shows a straight line having a constant slope of 18 passing through the point (0, 14) which is the y-intercept. This means the situation is not proportional. When the situation is proportional, a proportion can be used to solve problems. When the ratio of each y to its corresponding x in every ordered pair is not constant, the situation is not proportional, thus not allowing any further problems to be solved using proportions. For example, each hour costs \$18 to rent the four chairs and two umbrellas. The total cost also includes the \$14 charge for the sandcastle building tools. If the Maritz's spend 6 hours at the beach, a proportion cannot be set up and solved to find the cost for 6 hours including the cost of the sandcastle tools.



Use the two tables for the family's rental costs to find at what number of hours the two family's rental charges are the same.

Hours spent at beach, h	1	2	3	4	5	10
Rental cost – Jameson's	\$25	\$50	\$75	\$100	\$125	\$250
Rental cost – Maritz's	\$32	\$50	\$68	\$86	\$104	\$194

The rental charges for both families will be \$50 for 2 hours at the beach.

Reasonableness



Another way to verify this is to write an equation where the two values are equal to each other and solve for the value of x , the number of hours.

$$25x = 18x + 14$$

$$7x = 14$$

$$x = 2$$

The rental cost for the Jameson family is represented by the equation $y = 25x$, where x is the number of hours spent at the beach, and y is the total cost for their equipment rental. A table and a graph also represent the situation as shown in the solution process. The cost of the Jameson's rental is proportional to the number of hours spent at the beach.

Cluster 8.5: Proportionality

The rental cost for the Maritz family is represented by the equation $y = 18x + 14$, where x is the number of hours spent at the beach, and y is the total cost for their equipment rental. A table and a graph also represent the situation as shown in the solution process. The cost of the Maritz's rental is non-proportional to the number of hours spent at the beach.

If both families spend 2 hours at the beach, their rental costs are equal

Look For...

- two correct representations for each family's rental costs based on time spent at the beach including tables, graphs, and equations
- understanding of how to identify a situation as proportional or non-proportional
- correct identification of each family's situation as proportional or non-proportional
- determination of an equal total cost for the two families for 2 hours of time
- student justification of choices of solution strategy

Cluster 8.5: Proportionality

● Differentiation: Simplified Task

The Jameson family of 5 and the Maritz family of 4 are spending the day at the beach. There they find a vendor renting some items they might needed for the day.



The Jameson's decide to get chairs for each person in their family and to share 3 umbrellas with each other for their day on the beach. The Maritz family also gets chairs for each person as well as 2 umbrellas to share. The Maritz family also rents the sandcastle tools for the day.

How can the cost of each family's rentals be represented using a table, graph, or equation? Is each family's cost proportional to the number of hours they spend at the beach? Justify your reasoning.

Answer:

The Jameson family cost can be represented as $y = \$25x$ as an equation, table, or graph.

The Maritz family cost can be represented as $y = \$18x + 14$ as an equation, table, or graph.

The Jameson family's cost equation is proportional while the Maritz family's equation is not proportional.

■ Differentiation: Enriching Task

The Jameson family of 5 and the Maritz family of 4 are spending the day at the beach. There they find a vendor renting some items they might needed for the day.



The Jameson's decide to get chairs for each person in their family and to share 3 umbrellas with each other for their day on the beach. The Maritz family also gets chairs for each person as well as 2 umbrellas to share. ~~and~~ The Maritz family also rents the sandcastle tools for the day.

Use at least two representations, to represent the cost of each family's rentals. Is each family's cost proportional to the number of hours they spend at the beach? At how many hours would the two families' costs be the same? If each family spends 6 hours at the beach, how much will they each spend on rentals? If the Jameson's spent \$225, how long were they at the beach? If the Maritz family spent \$56 less than the Jameson's and both families were at the beach the same length of time, how many hours were the families at the beach? Justify your reasoning.

Answer:

The Jameson family cost can be represented as $y = \$25x$ as an equation, table, or graph.

The Maritz family cost can be represented as $y = \$18x + 14$ as an equation, table, or graph.

The Jameson family's cost equation is proportional while the Maritz family's equation is not proportional. The costs for the two family's rentals are the same at 2 hours. At 6 hours, the Jameson's cost is \$150 and the Maritz's cost is \$122. If the Jameson's spent \$225, they spent 9 hours at the beach. If the Maritz family spent \$56 less than the Jameson's, they were at the beach 10 hours.

Cluster 8.5: Proportionality

Scaffolded Task with Answers

The Jameson family of 5 and the Maritz family of 4 are spending the day at the beach. There they find a vendor renting some items they might need for the day.



The Jameson's decide to get chairs for each person in their family and to share 3 umbrellas with each other for their day on the beach. The Maritz family also gets chairs for each person as well as 2 umbrellas to share. The Maritz family also rents the sandcastle tools for the day.

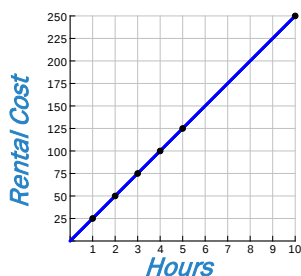
1. How much does the Jameson family spend on renting chairs per hour? How much does the Jameson family spend on renting umbrellas per hour?
chairs = \$13, umbrellas = \$12
2. How much does the Jameson family spend each hour for all their equipment?
\$25
3. Complete the table below to show the rental costs for the Jameson family based on the hours spent at the beach.

Hours spent at beach, h	1	2	3	4	5	10
Process	<i>$1 \bullet 25$</i>	<i>$2 \bullet 25$</i>	<i>$3 \bullet 25$</i>	<i>$4 \bullet 25$</i>	<i>$5 \bullet 25$</i>	<i>$10 \bullet 25$</i>
Rental cost, c	<i>\$25</i>	<i>\$50</i>	<i>\$75</i>	<i>\$100</i>	<i>\$125</i>	<i>\$250</i>

4. Write an equation to represent the Jameson's rental cost based on any number of hours spent at the beach.
 $y = 25x$

Cluster 8.5: Proportionality

5. Represent the Jameson's rental costs based on the number of hours spent at the beach on the graph provided. Label the graph appropriately and note the x- and y-axis values and scale used.



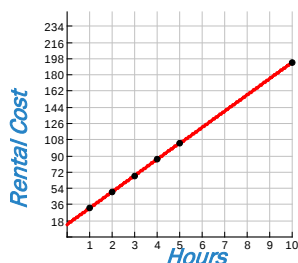
6. Is the Jameson family's rental situation proportional? Justify your reasoning.
Yes, because the rule forms a straight line, it has a constant ratio of y to x for each ordered pair in the table, and it passes through the origin. A proportion can be used to solve problems.
7. How much does the Maritz family spend on renting chairs per hour? How much does the Maritz family spend on renting umbrellas per hour?
chairs = \$10, umbrellas = \$8
8. How much does the Maritz family spend each hour for all their equipment?
\$18 for the hourly rented equipment
9. What other charges do the Maritz's have and how is it calculated?
They have a \$14 charge for the whole day that will be added on to the hourly rentals.
10. Complete the table below to show the rental costs for the Maritz family based on the hours spent at the beach.

Hours spent at beach, h	1	2	3	4	5	10
Process	$1 \bullet 18 + 14$	$2 \bullet 18 + 14$	$3 \bullet 18 + 14$	$4 \bullet 18 + 14$	$5 \bullet 18 + 14$	$10 \bullet 18 + 14$
Rental cost, c	\$32	\$50	\$68	\$86	\$104	\$194

11. Write an equation to represent the Maritz's rental cost based on any number of hours spent at the beach.
 $y = 18x + 14$

Cluster 8.5: Proportionality

12. Represent the Maritz's rental costs based on the number of hours spent at the beach on the graph provided. Label the graph appropriately and note the x- and y-axis values and scale used.



13. Is the Maritz family's rental situation proportional? Justify your reasoning.

No, because the ratio y to x in in the table for each ordered pair of values is not constant. The line is straight but the y -intercept is not at the origin. Also, a proportion cannot be used to solve problems.

14. At how many hours would the two families' costs be the same?

2 hours

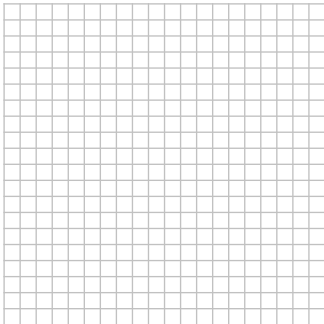
Performance Task: 8.5F
Proportional & Non-Proportional Relationships: Beach Rentals

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The Jameson's decide to get chairs for each person in their family and to share 3 umbrellas with each other for their day on the beach. The Maritz family also gets chairs for each person as well as 2 umbrellas to share. The Maritz family also rents the sandcastle tools for the day.

Use at least two representations, to represent the cost of each family's rentals. Is each family's cost proportional to the number of hours they spend at the beach? At how many hours would the two families' costs be the same? Justify your reasoning.



Procedural	0	1	2
Conceptual	0	1	2
Communication	0	1	2

Total points: _____

Performance Task: 8.5F
Proportional & Non-Proportional Relationships: Beach Rentals

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How can the cost of each family's rentals be represented using a table, graph, or equation? Is each family's cost proportional to the number of hours they spend at the beach? Justify your reasoning.

Procedural	0	1	2
Conceptual	0	1	2
Communication	0	1	2

Total points: _____

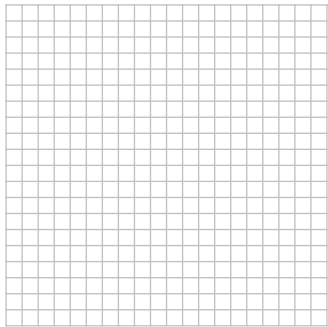
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Use at least two representations, to represent the cost of each family's rentals. Is each family's cost proportional to the number of hours they spend at the beach? At how many hours would the two families' costs be the same? If each family spends 6 hours at the beach, how much will they each spend on rentals? If the Jameson's spent \$225, how long were they at the beach? If the Maritz family spent \$56 less than the Jameson's and both families were at the beach the same length of time, how many hours were the families at the beach? Justify your reasoning.

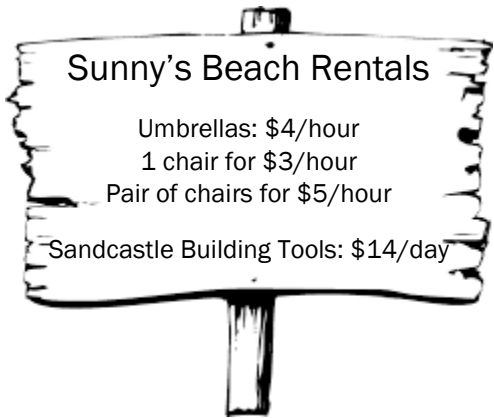


Procedural	0	1	2
Conceptual	0	1	2
Communication	0	1	2

Total points:_____

Performance Task: 8.5F
Proportional & Non-Proportional Relationships: Beach Rentals

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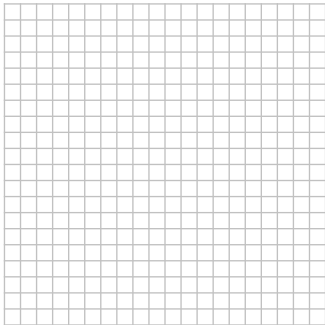
The Jameson's decide to get chairs for each person in their family and to share 3 umbrellas with each other for their day on the beach. The Maritz family also gets chairs for each person as well as 2 umbrellas to share. The Maritz family also rents the sandcastle tools for the day.

1. How much does the Jameson family spend on renting chairs per hour? How much does the Jameson family spend on renting umbrellas per hour?
2. How much does the Jameson family spend each hour for all their equipment?
3. Complete the table below to show the rental costs for the Jameson family based on the hours spent at the beach.

Hours spent at beach, h	1	2	3	4	5	10
Process						
Rental cost, c						

4. Write an equation to represent the Jameson's rental cost based on any number of hours spent at the beach.

5. Represent the Jameson’s rental costs based on the number of hours spent at the beach on the graph provided. Label the graph appropriately and note the x- and y-axis values and scale used.

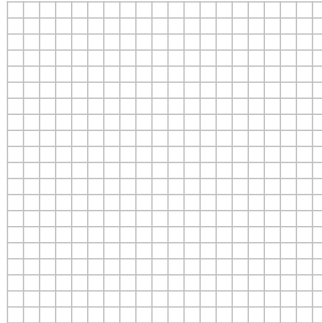


6. Is the Jameson family’s rental situation proportional? Justify your reasoning.
7. How much does the Maritz family spend on renting chairs per hour? How much does the Maritz family spend on renting umbrellas per hour?
8. How much does the Maritz family spend each hour for all their equipment?
9. What other charges do the Maritz’s have and how is it calculated?
10. Complete the table below to show the rental costs for the Maritz family based on the hours spent at the beach.
- | | | | | | | |
|---------------------------|---|---|---|---|---|----|
| Hours spent at beach, h | 1 | 2 | 3 | 4 | 5 | 10 |
| Process | | | | | | |
| Rental cost, c | | | | | | |
11. Write an equation to represent the Maritz’s rental cost based on any number of hours spent at the beach.



Name _____ Date _____

12. Represent the Maritz's rental costs based on the number of hours spent at the beach on the graph provided. Label the graph appropriately and note the x- and y-axis values and scale used.



13. Is the Maritz family's rental situation proportional? Justify your reasoning.

14. At how many hours would the two families' costs be the same?

