Name_____ Date _____

Distinguishing Between Proportional and Non-Proportional Relationships

Independent Practice

Directions: For each of the situations below, complete the tables. Answer the debriefing questions.

At Lake Sam Rayburn, there are two boat companies that rent boats for day use. Their schedule of charges is shown. Complete both tables, and use the process column to determine an expression that can be used to calculate the cost of renting a boat for *x* hours.

Cedar Bay Boat Rental
$25 per Hour
No Deposit

Hours	Process	Cost	$\frac{y}{x} = \frac{\text{Cost}}{\text{Hours}}$
0	25(0)	0	n/a
1	25(1)	25	$\frac{25}{1} = 25$
2			
3			
4			
x		*y*	

Pine Island Boats
$15 per Hour
$50 Deposit

Hours	Process	Cost	$\frac{y}{x} = \frac{\text{Cost}}{\text{Hours}}$
0	50 + 15(0)	50	n/a
1	50 + 15(1)	65	$\frac{65}{1} = 65$
2			
3			
4			
x		*y*	

Debriefing Questions

1. What patterns do you observe in the tables?

2. For each row of each table divide cost by hours. What do you notice?

3. What are the similarities and differences in the algebraic expressions for each company?

Name_____ Date _____

4. A **proportional relationship** is a relationship between two sets of numbers, x and y, such that the ratio of y to x is constant. Which of these two boat rental companies appears to use a proportional relationship to determine their charges?

For questions 5-8, determine if the table of values represents a proportional relationship. If so, then identify the constant of proportionality.

5

x	2	3	5	6
y	7.5	11.25	18.75	22.5

6

x	*y*
1	1
2	4
3	9
4	16

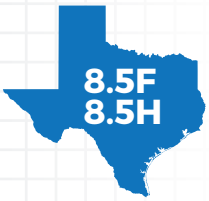
7

x	4	5	8	10
y	13	22	61	97

8

x	*y*
1	0.8
2	1.6
5	4
8	6.4

PROPORTIONAL AND NON-PROPORTIONAL RELATIONSHIPS



The student is expected to distinguish between proportional and non-proportional situations using tables, graphs, and equations in the form $y = kx$ or $y = mx + b$, where $b \neq 0$.

The student is expected to identify examples of proportional and non-proportional functions that arise from mathematical and real-world problems.

TELL ME MORE...

A **proportional relationship** is where paired values from the independent variable and dependent variable have a constant ratio. A linear relationship that is non-proportional is where the paired values do not have a constant ratio. In either case, both proportional relationships and non-proportional linear relationships have a constant rate of change of the dependent variable with respect to the independent variable.

PROPORTIONAL LINEAR RELATIONSHIPS

General form: $y = kx$

- k is the constant of proportionality
- k is equivalent to the constant rate of change

Attributes

- **Graph** contains the origin
- **Table** shows a constant ratio of paired values
- **Equation** does not have a constant term
- When $x = 0$, $y = 0$

NON-PROPORTIONAL LINEAR RELATIONSHIPS

General form: $y = mx + b$, where $b \neq 0$

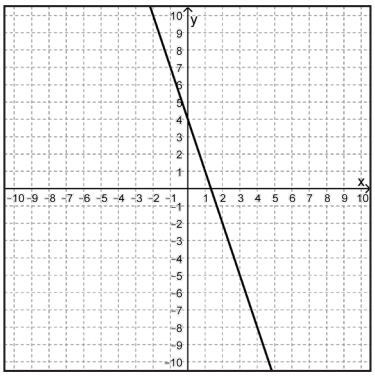
- m is the slope or constant rate of change
- b is the y-coordinate of the y-intercept

Attributes

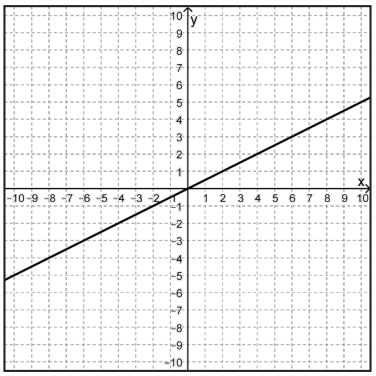
- **Graph** does not contain the origin
- **Table** shows a changing ratio of paired values
- **Equation** has a non-zero constant term
- When $x = 0$, $y = b$ ($b \neq 0$)

EXAMPLES

EXAMPLE 1: Which of the two graphs shown describes a proportional relationship? Which shows a non-proportional linear relationship? Explain your answer.



Graph I



Graph II

STEP 1 Both graphs show a line with a constant rate of change, so both relationships are linear. The graph of a proportional relationship passes through the origin, (0, 0). Identify the graph that contains (i.e., passes through) the origin.

Graph II shows a proportional relationship because the line contains the origin.

STEP 2 The graph of a non-proportional linear relationship has a constant rate of change but does not contain the origin. Identify the graph that has a constant rate of change and does not contain (i.e., pass through) the origin.

Graph I shows a non-proportional linear relationship because the line has a constant rate of change and does not contain the origin.

EXAMPLE 2: Which of the tables shows a proportional relationship? Which shows a non-proportional linear relationship?

Table 1

x	y
1	1.75
2	2
3	2.25
4	2.5
5	2.75

Table 2

x	y
0	0
1	1.5
2	6
3	13.5
4	24

Table 3

x	y
1	0.75
2	1.5
3	2.25
4	3
5	3.75

STEP 1 Analyze Table 1. Determine if it has a constant rate of change or a constant ratio of paired values.

	x	y	
	1	1.75	
2 – 1 = 1	2	2	2 – 1.75 = 0.25
3 – 2 = 1	3	2.25	2.25 – 2 = 0.25
4 – 3 = 1	4	2.5	2.5 – 2.25 = 0.25
5 – 4 = 1	5	2.75	2.75 – 2.5 = 0.25

Table 1 has a constant rate of change, $\frac{0.25}{1} = 0.25$, so it is linear.

x	y	$\frac{y}{x}$
1	1.75	$\frac{1.75}{1} = 1.75$
2	2	$\frac{2}{2} = 1$
3	2.25	$\frac{2.25}{3} = 0.75$
4	2.5	$\frac{2.5}{4} = 0.625$
5	2.75	$\frac{2.75}{5} = 0.55$

YOU TRY IT!

Determine whether the table shows a proportional relationship, non-proportional linear relationship, or neither. Justify your answer.

x	y
2	79
4	83
6	87
8	91
10	95

Constant rate of change? _____

Constant ratio? _____

Relationship: _____

Table 1 does not have a constant ratio of paired values so it is not proportional.

Table 1 shows a non-proportional linear relationship.

STEP 2 Analyze Table 2. Determine if it has a constant rate of change or a constant ratio of paired values.

x	y	
0	0	
		$1 - 0 = 1$ and $1.5 - 0 = 1.5$
1	1.5	
		$2 - 1 = 1$ and $6 - 1.5 = 4.5$
2	6	
		$3 - 2 = 1$ and $13.5 - 6 = 7.5$
3	13.5	
		$4 - 3 = 1$ and $24 - 13.5 = 10.5$
4	24	

Table 2 does not have constant rate of change so it is not linear.

Table 2 shows neither a proportional nor a non-proportional linear relationship.

STEP 3 Analyze Table 3. Determine if it has a constant rate of change or a constant ratio of paired values.

x	y	
1	0.75	
		$2 - 1 = 1$ and $1.5 - 0.75 = 0.75$
2	1.5	
		$3 - 2 = 1$ and $2.25 - 1.5 = 0.75$
3	2.25	
		$4 - 3 = 1$ and $3 - 2.25 = 0.75$
4	3	
		$5 - 4 = 1$ and $3.75 - 3 = 0.75$
5	3.75	

Table 3 has a constant rate of change.

Table 3 shows a proportional relationship.

x	y	$\frac{y}{x}$
1	0.75	$\frac{0.75}{1} = 0.75$
2	1.5	$\frac{1.5}{2} = 0.75$
3	2.25	$\frac{2.25}{3} = 0.75$
4	3	$\frac{3}{4} = 0.75$
5	3.75	$\frac{3.75}{5} = 0.75$

Table 3 has a constant ratio of paired values so it is proportional.

EXAMPLE 3: Does the situation shown represent a proportional or non-proportional linear relationship? How do you know?

The cost of purchasing n gallons of gasoline is \$2.19 per gallon plus a fuel tax of \$0.35 per gallon.

STEP 1 Write an equation to represent the situation.

- The cost of 1 gallon of gasoline is \$2.19(1). The cost of 2 gallons of gasoline is \$2.19(2). The cost of n gallons of gasoline is \$2.19n.
- The fuel tax for 1 gallon of gasoline is \$0.35(1). The fuel tax for 2 gallons of gasoline is \$0.35(2). The fuel tax for n gallons of gasoline is \$0.35n.
- Let C represent the combined cost of the gasoline and fuel tax for n gallons of gasoline.
- $C = 2.19n + 0.35n$

C* = 2.54*n

STEP 2 Determine whether the equation is of the form $y = kx$ for a proportional relationship or $y = mx + b$ where $b \neq 0$ for a non-proportional linear relationship.

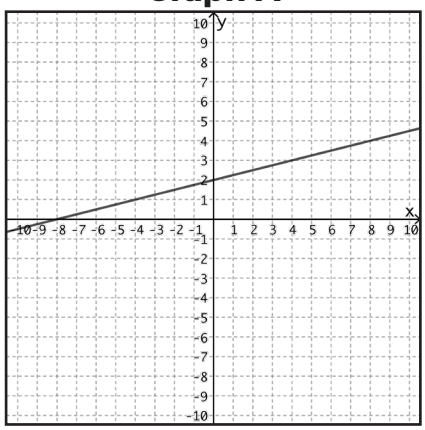
- The equation, $C = 2.54n$, has no constant term.
- The equation matches the form $y = kx$.

The situation represents a proportional relationship.

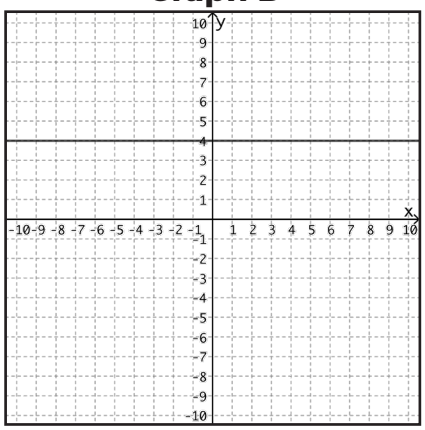
PRACTICE

Use the following graphs to answer questions 1–2.

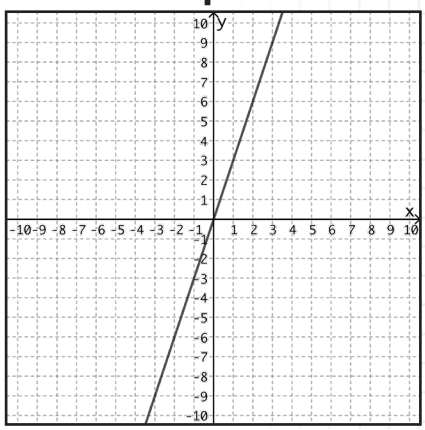
Graph A



Graph B



Graph C



1. Given the graphs shown above, which graph(s) show(s) a proportional relationship?

2. Given the graphs shown above, which graph(s) show(s) a non-proportional relationship?

Use the following tables to answer questions 3-4.

Table A

x	y
1	6
2	7
3	8
4	9

Table B

x	y
3	1
12	4
18	6
33	11

Table C

x	y
8	8
14	12.5
28	23
44	35

Table D

x	y
4	24
13	78
25	150
39	234

3. Given the tables shown above, which table(s) show(s) a proportional relationship?

4. Given the tables shown above, which table(s) show(s) a non-proportional relationship?

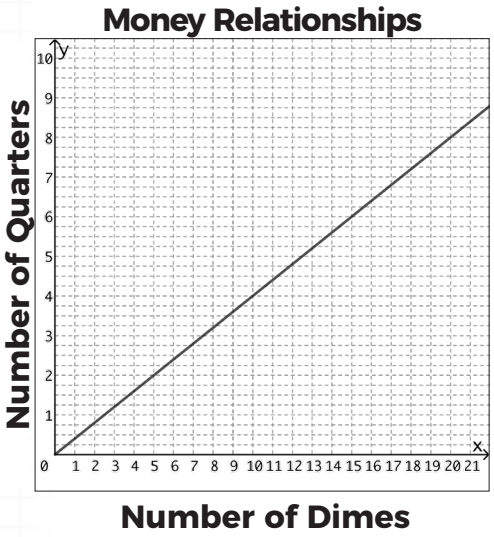
Use the following equations to answer questions 5-6.

Equation A	Equation B	Equation C	Equation D	Equation E
$y = -2$	$y = \frac{5}{2}x$	$y = \frac{2}{3}x + 4$	$y = 3x + 5$	$y = 4x$

5. Given the equations shown above, which equation(s) show(s) a proportional relationship?

6. Given the equations shown above, which equation(s) show(s) a non-proportional relationship?

7. The graph below shows the relationship between the number of dimes and the number of quarters that produce an equivalent amount of money. Is the relationship shown in the graph a proportional or non-proportional relationship? Explain how you know.



8. The table below shows costs for a person attending the local carnival, including the price of ride and food tickets. Based on the information in the table, is the relationship between cost and number of tickets purchased proportional or non-proportional? Explain how you know.

Tickets Purchased, x	Total Cost, y
10	$12.50
16	$20.00
28	$35.00
50	$62.50

9. Aaron uses the equation $y = 500 + 0.2x$ to determine his weekly paycheck based on his sales volume. Does Aaron's equation represent a proportional or a non-proportional relationship? Explain how you know.

10. Which of the following situations represents a proportional relationship?

A The amount Andrew earns mowing lawns for $18 per hour.

B The cost of baseball tickets online at $7 per ticket plus a $3 online processing fee.

C The total cost of a taxi that charges $2 per mile plus a passenger fee of $1.50 to pick up a rider.

D The height of a 10-inch candle after burning at a rate of $\frac{3}{4}$-inch each hour.

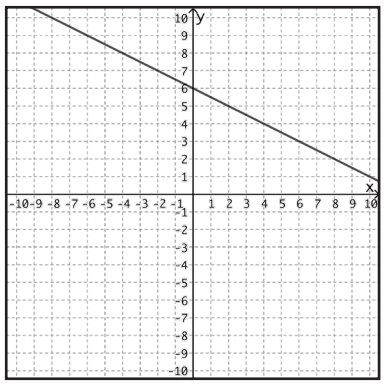
11. Which of the following representations shows a non-proportional relationship?

F

x	y
17	51
21	63
26	78
36	108

G $y = \frac{3}{8}x$

H



J The number of cans of sodas in x packages of 12 each.

Name _____ Date _____

Performance Task: 8.5F
Proportional & Non-Proportional Relationships: Beach Rentals

The Jameson family of 5 and the Maritz family of 4 are spending the day at the beach. There they find a vendor renting some items they might need for the day.

The Jameson's decide to get chairs for each person in their family and to share 3 umbrellas with each other for their day on the beach. The Maritz family also gets chairs for each person as well as 2 umbrellas to share. The Maritz family also rents the sandcastle tools for the day.

Use at least two representations, to represent the cost of each family's rentals. Is each family's cost proportional to the number of hours they spend at the beach? At how many hours would the two families' costs be the same? Justify your reasoning.

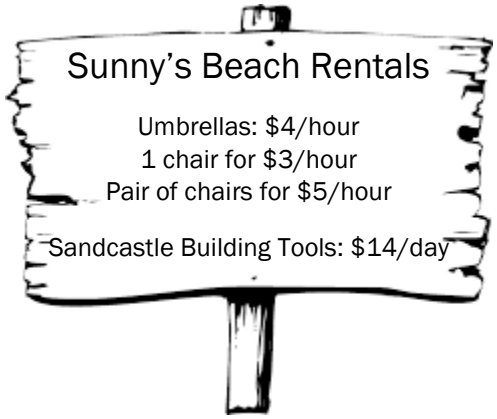
Procedural	0	1	2
Conceptual	0	1	2
Communication	0	1	2

Total points:_____

Name _____ Date _____

Performance Task: 8.5F
Proportional & Non-Proportional Relationships: Beach Rentals

The Jameson family of 5 and the Maritz family of 4 are spending the day at the beach. There they find a vendor renting some items they might need for the day.



The Jameson's decide to get chairs for each person in their family and to share 3 umbrellas with each other for their day on the beach. The Maritz family also gets chairs for each person as well as 2 umbrellas to share. The Maritz family also rents the sandcastle tools for the day.

1. How much does the Jameson family spend on renting chairs per hour? How much does the Jameson family spend on renting umbrellas per hour?

2. How much does the Jameson family spend each hour for all their equipment?

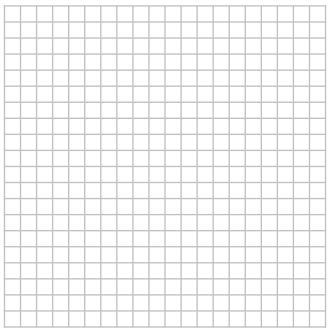
3. Complete the table below to show the rental costs for the Jameson family based on the hours spent at the beach.

Hours spent at beach, *h*	1	2	3	4	5	10
Process						
Rental cost, *c*						

4. Write an equation to represent the Jameson's rental cost based on any number of hours spent at the beach.

Name _____ Date _____

5. Represent the Jameson's rental costs based on the number of hours spent at the beach on the graph provided. Label the graph appropriately and note the *x*- and *y*-axis values and scale used.



6. Is the Jameson family's rental situation proportional? Justify your reasoning.

7. How much does the Maritz family spend on renting chairs per hour? How much does the Maritz family spend on renting umbrellas per hour?

8. How much does the Maritz family spend each hour for all their equipment?

9. What other charges do the Maritz's have and how is it calculated?

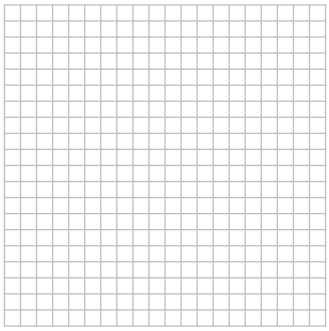
10. Complete the table below to show the rental costs for the Maritz family based on the hours spent at the beach.

Hours spent at beach, *h*	1	2	3	4	5	10
Process						
Rental cost, *c*						

11. Write an equation to represent the Maritz's rental cost based on any number of hours spent at the beach.

Name _____ Date _____

12. Represent the Maritz's rental costs based on the number of hours spent at the beach on the graph provided. Label the graph appropriately and note the *x*- and *y*-axis values and scale used.



13. Is the Maritz family's rental situation proportional? Justify your reasoning.

14. At how many hours would the two families' costs be the same?

Name _____ Date _____

Performance Task: 8.5F

Proportional & Non-Proportional Relationships: Beach Rentals

The Jameson family of 5 and the Maritz family of 4 are spending the day at the beach. There they find a vendor renting some items they might need for the day.

The Jameson's decide to get chairs for each person in their family and to share 3 umbrellas with each other for their day on the beach. The Maritz family also gets chairs for each person as well as 2 umbrellas to share. The Maritz family also rents the sandcastle tools for the day.

How can the cost of each family's rentals be represented using a table, graph, or equation? Is each family's cost proportional to the number of hours they spend at the beach? Justify your reasoning.

Procedural	0	1	2
Conceptual	0	1	2
Communication	0	1	2

Total points:_____

Name _____ Date _____

Performance Task: 8.5F
Proportional & Non-Proportional Relationships: Beach Rentals

The Jameson family of 5 and the Maritz family of 4 are spending the day at the beach. There they find a vendor renting some items they might need for the day.

The Jameson's decide to get chairs for each person in their family and to share 3 umbrellas with each other for their day on the beach. The Maritz family also gets chairs for each person as well as 2 umbrellas to share. The Maritz family also rents the sandcastle tools for the day.

Use at least two representations, to represent the cost of each family's rentals. Is each family's cost proportional to the number of hours they spend at the beach? At how many hours would the two families' costs be the same? If each family spends 6 hours at the beach, how much will they each spend on rentals? If the Jameson's spent $225, how long were they at the beach? If the Maritz family spent $56 less than the Jameson's and both families were at the beach the same length of time, how many hours were the families at the beach? Justify your reasoning.

Procedural	0	1	2
Conceptual	0	1	2
Communication	0	1	2

Total points:_____