Cluster 8.5: Proportionality

8.5H: Proportional & Non-Proportional Relationships: Shipping Plans

Focusing TEKS

8.5H Proportionality. The student applies mathematical process standards to use proportional and non-proportional relationships to develop foundational concepts of functions. The student is expected to identify proportional and non-proportional functions that arise from mathematical and real-world problems. **Supporting Standard**

Additional TEKS:

8.5A Represent linear proportional situations with tables, graphs, and equations in the form of $y = kx$. **Supporting Standard**

8.5B Represent linear non-proportional situations with tables, graphs, and equations in the form of $y = mx + b$, where $b \neq 0$. **Supporting Standard**

8.5F Distinguish between proportional and non-proportional situations using tables, graphs, and equations in the form of $y = kx$ or $y = mx + b$, where $b \neq 0$. **Supporting Standard**

7.4C Determine the constant of proportionality ($k = y/x$) within mathematical and real-world problems. **Supporting Standard**

Focusing Mathematical Process

8.1A Apply mathematics to problems arising in everyday life, society, and the workplace.

8.1B Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution.

8.1D Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

8.1F Analyze mathematical relationships to connect and communicate mathematical ideas.

Performance Task

Mrs. Mancini is starting a small business where she will ship her product direct to customers. She wants to make sure her shipping and handling charges are proportional to the amount of the customer's purchase. Mrs. Mancini is trying to select among 3 different shipping plans:

Plan A: charges a flat rate of \$10 plus 5% of the order's subtotal for orders over \$100;
Plan B: charges 15% of the order's subtotal for shipping regardless of the order's size;
Plan C: uses a graduated scale of \$10 shipping for orders totaling ≤ \$100, \$20 for orders totaling \$100 < total ≤ \$200, \$30 for orders totaling \$200 < total ≤ \$300, etc.

Which plan(s) meet(s) her needs? Justify your reasoning.

Answer: *Plan B is a proportional shipping charge dependent on the subtotal spent.*

Cluster 8.5: Proportionality

Mathematically Speaking...

In this task, students review three options for calculation of shipping charges a new business owner could use and determine which plan offers a proportional shipping charge based on the customer's order amount. Students are expected to use representations to model and determine which plan represents a proportional function. One or multiple representations, including equation, table, and/or graph forms, could be used to demonstrate proportionality.

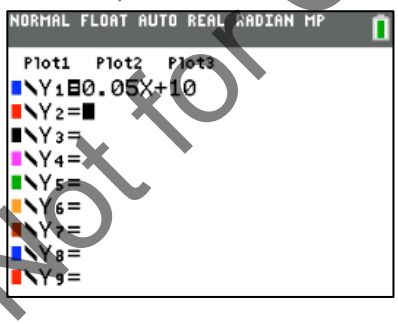
Students begin the foundation for proportions in grade 5 as they explore multiplicative relationships. Once students enter grade 6 mathematics, they work with multiplicative comparisons, rates, and proportions as a means to solve problems. Students continue this work throughout grade 7 when discovering the constant of proportionality, and with writing linear equations. In this task students must apply this knowledge to distinguish between proportional and non-proportional situations.

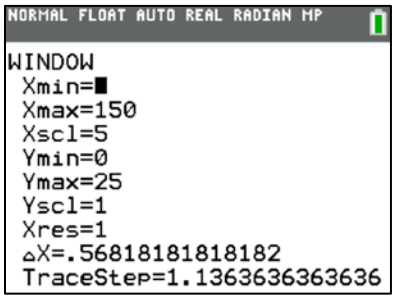
Possible Solution

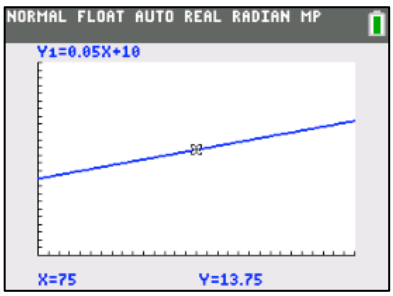
When a situation is proportional, the ratio of each output to its corresponding input is a constant, the ordered pairs of values that satisfy the function rule describing the situation are linear, and the origin is included in the set of possible ordered pairs. To determine which plan is proportional, use a representation such as an equation, a table, or a graph to examine if the definition of a proportional relationship is exists.

Plan A: This plan has Mrs. Mancini charging her customers $10 plus 5% of their order's total. First, use an equation to model the situation. The order subtotal can be represented using a variable such as x, and the output variable y represents the total amount of shipping charged to the customer. To calculate the shipping charge to a customer under plan A, take the order subtotal times 5%, and since the $10 fee is charged regardless of size of order, $10 is added after the percentage is calculated. The equation for Plan A is $y = 5\%(x) + 10$ or $y = 0.05x + 10$.

Using a graphing calculator, enter the equation into the equation editor, and graph the equation using an appropriate viewing window.



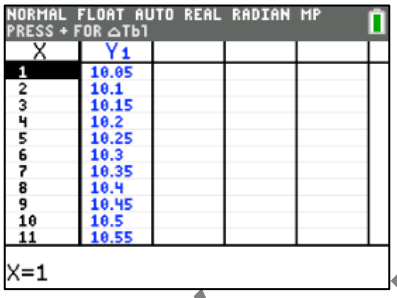




The price of the shipping for a package increases as the order subtotal increases. Because the graph is a straight line with a positive slope, the rate of change between ordered pairs is constant.

Cluster 8.5: Proportionality

Next, determine if the ratio of each output to its corresponding input is a constant. Use the table of values in the graphing calculator for a list of ordered pairs.



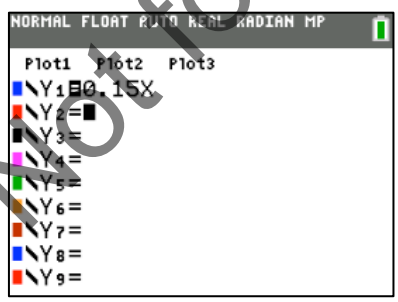
For several ordered pairs of values, create a ratio of the output (Y_1) to the input (x). Simplify each ratio to a common form to determine if the ratios comparing the x and y coordinates of the points are the same.

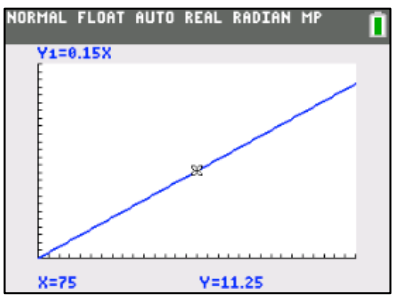
x	y	Ratio
1	10.05	$\frac{10.05}{1} = 10.05$
2	10.1	$\frac{10.1}{2} = 5.05$
4	10.2	$\frac{10.2}{4} = 2.55$
10	10.5	$\frac{10.5}{10} = 1.05$

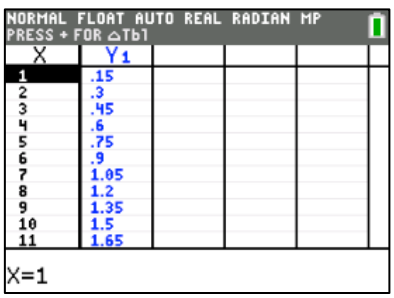
From the table, it is clear that the ratio of each y to its corresponding x is not constant. In a proportion, the ratio of y to x, or k, is always constant for all ordered pairs. While the situation has a constant rate of change, it does not have a constant of proportionality. Because the line has a y-intercept or starting value of 10 due to the flat fee of $10, the shipping charge is not proportional to the order amount.

Plan B: This plan has Mrs. Mancini charging her customers 15% of the order's total. First, use an equation to model the situation. The order subtotal can be represented using a variable such as x, and the output variable y represents the total amount of shipping charged to the customer. To calculate the shipping charge to a customer under plan B, take the order subtotal times 15%, regardless of size of order. The equation for Plan B is $y = 15\%(x)$ or $y = 0.15x$.

Using a graphing calculator, enter the equation into the equation editor, and graph the equation using an appropriate viewing window.







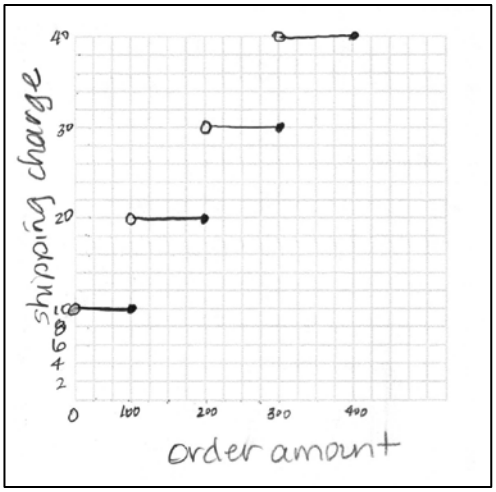
Cluster 8.5: Proportionality

The price of the shipping for a package increases as the order subtotal increases. Because the graph is a straight line with a positive slope, the rate of change between ordered pairs is constant. Next, determine if the ratio of each output to its corresponding input is a constant. Use the table of values in the graphing calculator for a list of ordered pairs.

For several ordered pairs, create a ratio of the output (Y_1) to the input (x). Simplify each ratio to a common form to determine if the ratios of the points are the same.

x	y	Ratio
1	0.15	$\frac{0.15}{1} = 0.15$
2	0.3	$\frac{0.3}{2} = 0.15$
4	0.6	$\frac{0.6}{4} = 0.15$
10	1.5	$\frac{1.5}{10} = 0.15$

From the table, it is clear that the ratio of each y to its corresponding x is constant. In a proportion, the ratio of y to x, or k, is always constant for all ordered pairs. Plan B has a constant rate of change and a constant of proportionality. Because the line has a y-intercept or starting value of 0, the origin is also part of the possible ordered pairs in the solution. Therefore, the shipping charge is proportional to the order amount.



Plan C: This plan has Mrs. Mancini charging her customers based on a scale using their order amount. For the scale, if a customer spends ≤ \$100, the shipping charge is a flat fee of \$10 shipping regardless of the specific order size. If a customer spends \$100 < total ≤ \$200 the shipping fee is \$20. And if the customer spends \$200 < total ≤ \$300 the shipping fee is \$30. This fee plan continues for each group of \$100 spent on the order. This situation cannot be modeled with a single equation. Rather this situation is a series of inequality statements. If plotted, the y-values corresponding to the x amounts spent between 0 through 100 all are the same at 10, or a horizontal line segment. The y-values that correspond to an x between 10 up through 200 would all be the same at 20, forming another horizontal segment. The graph would appear as a series of horizontal line segments placed 10 units apart on the y-axis every 100 units on the x-axis.

Cluster 8.5: Proportionality

The price of the shipping for a package increases as the order subtotal increases, but not in a linear or constant manner. Next, determine if the ratio of each output to its corresponding input is a constant value. Use a table of possible values for ordered pairs, and create a ratio of the output (y) to the input (x). Simplify each ratio to a common form to determine if the ratios of the points are the same.

x	y	Ratio
1	10	$\frac{10}{1}$ = 10
100	10	$\frac{100}{10}$ = 10
210	20	$\frac{210}{20}$ = 10.5
350	30	$\frac{350}{30}$ = 11.67

From the table, it is clear that the ratio of each y to its corresponding x is not constant for all possible ordered pairs. In a proportion, the ratio of y to x, or k, is always constant for all ordered pairs. While the situation forms some line segments, it is not linear, and it does not have a constant of proportionality. Plan C is not proportional to the order amount.

Of the 3 shipping charge plans, Plan B is the only one that charges a shipping amount that is proportional to the order's amount spent by the customer.

Look For...

- use of appropriate representations including equation, graph, and/or table to evaluate each plan for proportionality
- understanding of how to identify a situation as proportional or non-proportional
- correct identification of each plan as proportional or non-proportional
- student justification of choices of solution strategy

Cluster 8.5: Proportionality

🔵 Differentiation: Simplified Task	🟦 Differentiation: Enriching Task
Mrs. Mancini is starting a small business where she will ship her product direct to customers. She wants to make sure her shipping and handling charges are proportional to the amount of the customer's purchase. Mrs. Mancini is trying to select between 2 different shipping plans: **Plan A**: charge a flat rate of $10 plus 5% of the order subtotal **Plan B**: charge 15% of the order subtotal for shipping regardless of order size Which plan meets her needs? Justify your reasoning. ***Answer:*** *Plan B is a proportional shipping charge dependent on the subtotal spent.*	Mrs. Mancini is starting a small business where she will ship her product direct to customers. She wants to make sure her shipping and handling charges are proportional to the amount of the customer's purchase. Mrs. Mancini is trying to select among 3 different shipping plans: **Plan A**: charges a flat rate of $10 plus 5% of the order's subtotal for orders over $100; **Plan B**: charges 15% of the order's subtotal for shipping regardless of the order's size; **Plan C**: uses a graduated scale of $10 shipping for orders totaling ≤ $100, $20 for orders totaling $100 < total ≤ $200, $30 for orders totaling $200 < total ≤ $300, etc. Which plan(s) meet(s) Mrs. Mancini's shipping needs based on the subtotal of the items to be shipped? Mrs. Mancini also thinks she wants to purchase a shipping plan with a carrier whose charges are proportional to the weight of the box shipped. The companies and their charges are shown here. Company A: $0.57 per pound Company B: $0.46 per pound plus $0.12 per pound over 20 pounds Company C: $250 per month for up to 30 boxes shipped and a total of no more than 400 combined pounds Which plan(s) meet(s) Mrs. Mancini's shipping needs based on the weight of the items being shipped? Justify your reasoning. ***Answer:*** *Plan B is a proportional shipping charge dependent on the subtotal spent. Company A is a proportional plan based on the weight of boxes being shipped.*

Cluster 8.5: Proportionality

Scaffolded Task with Answers

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1. If the customer's subtotal is represented as the input, *x*, and the shipping charge is the output, *y*, what equation can be used to determine the amount of the shipping charge for Plan A?
   y = 0.05x + 10

2. When graphed, what are the attributes of the equation for calculating shipping charges using Plan A?
   The graph is a linear function (straight line) with a slope of 0.05 and a y-intercept of 10.

3. For every possible ordered pair of input and output values using Plan A, would the ratio of *y* to *x* always be constant?
   No, the y-intercept value would need to be 0 to be proportional so there is not a constant of proportionality.

4. Does Plan A represent a shipping charge that is proportional to the amount of the customer's order?
   No

5. If the customer's subtotal is represented as the input, *x*, and the shipping charge is the output, *y*, what equation can be used to determine the amount of the shipping charge for Plan B?
   y = 0.15x

6. When graphed, what are the attributes of the equation for calculating shipping charges using Plan B?
   The graph is a linear function (straight line) with a slope of 0.15 and a y-intercept of 0.

7. For every possible ordered pair of input and output values using Plan B, would the ratio of *y* to *x* always be constant?
   Yes, the situation is linear and passes through the origin making the situation proportional with a constant of proportionality of 0.15.

8. Does Plan B represent a shipping charge that is proportional to the sub-total of the customer's order?
   Yes

9. If the customer's subtotal is represented as the input, *x*, and the shipping charge is the output, *y*, what equation can be used to determine the amount of the shipping charge for Plan C?
   It is not possible to write a single equation for the price of shipping for Plan C.

10. When graphed, what are the attributes of the equation for calculating shipping charges using Plan C?
    It forms a series of line segments representing inequalities along different y values.

11. For every possible ordered pair of input and output values using Plan C, would the ratio of *y* to *x* always be constant?
    No, the situation is not linear and does not pass through the origin, every ratio of y compared to x is not constant and the situation is not proportional.

12. Does Plan C represent a shipping charge that is proportional to the sub-total of the customer's order?
    No

13. Which plan(s) meet(s) Mrs. Mancini's needs?
    Plan B only

Name _____ Date _____

Performance Task: 8.5H

Proportional & Non-Proportional Relationships: Shipping Plans

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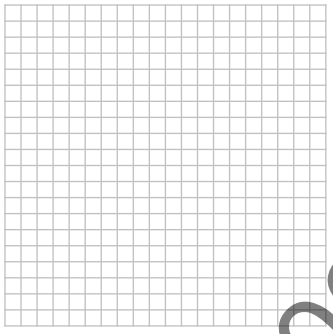
Which plan(s) meet(s) her needs? Justify your reasoning.



Procedural	0	1	2
Conceptual	0	1	2
Communication	0	1	2

Total points:_____

Name _____ Date _____

Performance Task: 8.5H

Proportional & Non-Proportional Relationships: Shipping Plans

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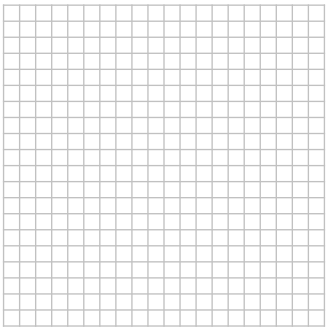

Procedural	0	1	2
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Name _____ Date _____

Performance Task: 8.5H

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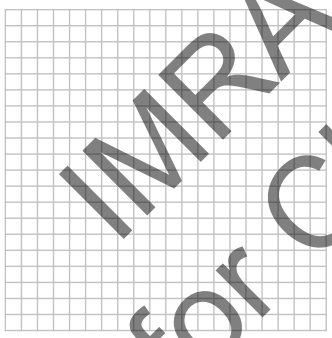
Which plan(s) meet(s) Mrs. Mancini's shipping needs based on the weight of the items being shipped? Justify your reasoning.



Procedural	0	1	2
Conceptual	0	1	2
Communication	0	1	2

Total points:_____

Name _____ Date _____

Performance Task: 8.5H
Proportional & Non-Proportional Relationships: Shipping Plans

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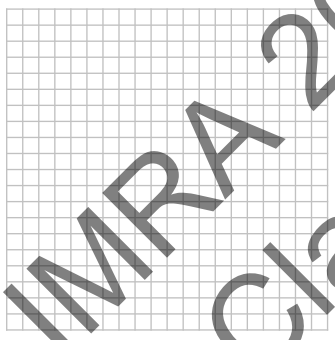
1. If the customer's subtotal is represented as the input, *x*, and the shipping charge is the output, *y*, what equation can be used to determine the amount of the shipping charge for Plan A?

2. When graphed, what are the attributes of the equation for calculating shipping charges using Plan A?



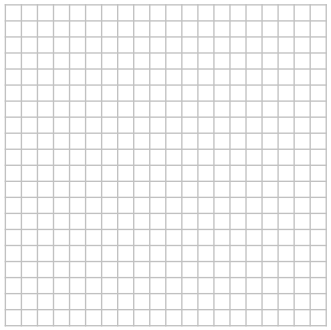
3. For every possible ordered pair of input and output values using Plan A, would the ratio of *y* to *x* always be constant?

4. Does Plan A represent a shipping charge that is proportional to the amount of the customer's order?

Name _____ Date _____

5. If the customer's subtotal is represented as the input, *x*, and the shipping charge is the output, *y*, what equation can be used to determine the amount of the shipping charge for Plan B?

6. When graphed, what are the attributes of the equation for calculating shipping charges using Plan B?

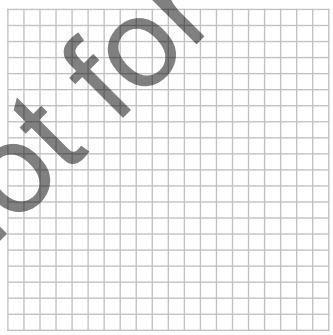


7. For every possible ordered pair of input and output values using Plan B, would the ratio of *y* to *x* always be constant?

8. Does Plan B represent a shipping charge that is proportional to the sub-total of the customer's order?

9. If the customer's subtotal is represented as the input, *x*, and the shipping charge is the output, *y*, what equation can be used to determine the amount of the shipping charge for Plan C?

10. When graphed, what are the attributes of the equation for calculating shipping charges using Plan C?



Name _____ Date _____

11. For every possible ordered pair of input and output values using Plan C, would the ratio of *y* to *x* always be constant?

12. Does Plan C represent a shipping charge that is proportional to the sub-total of the customer's order?

13. Which plan(s) meet(s) Mrs. Mancini's needs?