Cluster 6.4: Proportionality

6.4G: Rates and Ratios: After School Time

Focusing TEKS

6.4G Proportionality. The student applies mathematical process standards to develop an understanding of proportional relationships in problem situations. The student is expected to generate equivalent forms of fractions, decimals, and percents using real-world problems, including problems that involve money. **Readiness Standard**

Additional TEKS:

6.2E Extend representations for division to include fraction notation such as $\frac{a}{b}$ represents the same number as $a \div b$ where $b \neq 0$. **Supporting Standard**

6.3E Multiply and divide positive rational numbers fluently. **Readiness Standard**

6.4B Apply qualitative and quantitative reasoning to solve prediction and comparison of real-world problems involving ratios and rates. **Readiness Standard**

6.4E Represent ratios and percents with concrete models, fractions, and decimals. **Supporting Standard**

6.5B Solve real-world problems to find the whole given a part and the percent, to find the part given the whole and the percent, and to find the percent given the part and the whole, including the use of concrete and pictorial models. **Readiness Standard**

6.5C Use equivalent fractions, decimals, and percents to show equal parts of the same whole. **Supporting Standard**
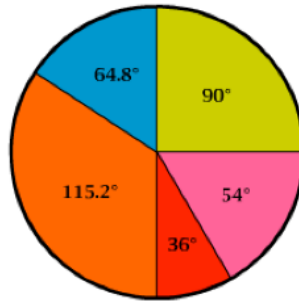
Focusing Mathematical Process

6.1A Apply mathematics to problems arising in everyday life, society, and the workplace.

6.1B Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution.

6.1C Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems.

6.1F Analyze mathematical relationships to connect and communicate mathematical ideas.

Cluster 6.4: Proportionality

⚠️ Performance Task

Students in Study Skills class completed a survey about after school time by tracking the amount of time spent in different after school activities for a week. The teacher combined all the information from her students and produced the following graph.

The *homework* section is 1.5 times larger than the *other activities* section. The largest amount of time after school is spent on *sports activities*. The fraction of time spent *watching TV* is 0.07 more than the fraction of time spent *playing video games*. What fraction of all after school time is spent on each activity in the graph? What percent of after school time does each sector of the graph represent? If the average student has 7 hours of after school time before going to bed, what portion of this time is spent on each activity? Justify your reasoning.

Answer: *The 115.2° sector is Sports Activities with $\frac{8}{25}$ or 32% of time. The 64.8° sector is Playing Video Games with $\frac{9}{50}$ or 18% of time. The 90° sector is Watching TV with $\frac{1}{4}$ or 25% of time. The 54° sector is Homework with $\frac{3}{20}$ or 15% of time. The 36° sector is Other Activities with $\frac{1}{10}$ or 10% of time. Of a 7-hour period of after school time, the average student would spend 2.24 hours in Sports Activities, 1.75 hours Watching TV, 1.26 hours Playing Video Games, 1.05 hours doing Homework, and 0.7 hours doing Other Activities.*

Mathematically Speaking…

In this task, students use a circle graph with given degrees marked for each sector and information about the after school activities represented in the sectors. Students will determine the activity that matches each sector and will use equivalent forms to determine the fractional portion and percent each sector represents on the graph. Once eachnpercent is determined, students will calculate the amount of hours each activity comprises of an average 7-hour after school period of time.

In solving, students can use various numeric techniques including division and equivalent fractions or can use models, or other strategies to represent and convert each of the ratios.

This task builds upon students' understanding of ratio as a proportion of a whole and connects to multiplication and division. Students use a variety of operations and strategies to relate equal parts of the whole. Students will also apply previous understanding of the degrees in a circle and the degrees in a central angle developed in Grade 4.

Cluster 6.4: Proportionality

Possible Solution

Use the information about the size relationships of each sector to determine which sector of the graph represents which activity. The activities are: Homework, Other Activities, Sports Activities, Watching TV, and Playing Video Games.

The Homework sector is 1.5 larger than the Other Activities sector. Using reasoning and mental math, the two sectors that appear to have this relationship are the sector marked 36° and the sector marked 54°. Check using a fraction and/or division to be sure the ratio is correct.

$$\frac{54°}{36°} = \frac{54 \div 18}{36 \div 18} = \frac{3}{2}$$

Because the simplified ratio is 3 to 2, and 3 ÷ 2 = 1.5, this means the 54° sector is 1.5 times larger than the 36° sector. The Homework sector is marked 54° and the Other Activities sector is marked 36°.

The problem states that the largest amount of time is spent on Sports Activities. This means the sector marked 115.2° represents Sports Activities.

The remaining two sectors represent Watching TV and Playing Video Games. The problem states that Watching TV is more than Playing Video Games because their fractional values are 0.07 or $\frac{7}{100}$ apart in size. Because this is known, the larger of the remaining two sectors, 90°, represents Watching TV and the smaller of the remaining sectors, 64.8°, represents Playing Video Games.

Convert each sector of the graph from the number of degrees in the circle to a fraction of the circle. A circle has 360 degrees. Each sector is a fraction of the total circle. To find the fraction for each sector, make a ratio of the number of degrees in the sector to the total degrees in the circle. Then simplify each fraction as necessary.

<u>Homework</u>

$$\frac{54°}{360°} = \frac{54 \div 18}{360 \div 18} = \frac{3}{20}$$

The Homework sector represents $\frac{3}{20}$ of all after school time. To convert this number to a percent, one way is to divide the numerator by the denominator to convert to a decimal value and then multiply times 100 since a percent is a ratio out of 100. Another way is to find a fraction equivalent with 100 as the denominator since a percent is a ratio out of 100.

$$\frac{3}{20} = \frac{3 \times 5}{20 \times 5} = \frac{15}{100}$$

Homework represents 15% of all after school time.

<u>Other:</u>

Because the Homework sector of the graph is 1.5 times the amount of the Other Activities sector, one way to find the Other sector is to use an equation.

Cluster 6.4: Proportionality

$$\text{Other Activities} \times \frac{3}{2} = \frac{3}{20}$$

To find the fraction for Other Activities, solve the equation by multiplying $\frac{3}{20}$ times the reciprocal of $\frac{3}{2}$.

$$\frac{3}{20} \times \frac{2}{3} = \frac{6}{60} = \frac{1}{10}$$

Reasonableness

Verify the fraction using the ratio of the degrees in the Other Activities sector to the total circle.

$$\frac{36°}{360°} = \frac{36 \div 36}{360 \div 36} = \frac{1}{10}$$

Convert the fraction $\frac{1}{10}$ to a value out of 100 to find the percent of the circle graph used for the Other Activities sector.

$$\frac{1}{10} = \frac{1 \times 10}{10 \times 10} = \frac{10}{100}$$

Other Activities represent 10% of all after school time.

<u>Watching TV</u>

This sector is 90° of the circle.

$$\frac{90°}{360°} = \frac{90 \div 90}{360 \div 90} = \frac{1}{4}$$

The Watching TV sector represents $\frac{1}{4}$ of all after school time. To convert this number to a percent, find a fraction equivalent with 100.

$$\frac{1}{4} = \frac{1 \times 25}{4 \times 25} = \frac{25}{100}$$

Watching TV represents 25% of all after school time.

<u>Playing Video Games:</u>

$$\frac{64.8°}{360°} = \frac{64.8 \times 10}{360 \times 10} = \frac{648}{3,600} = \frac{648 \div 72}{3,600 \div 72} = \frac{9}{50}$$

The Playing Video Games sector represents $\frac{9}{50}$ of all after school time. To convert this number to a percent, find a fraction equivalent with 100 as the denominator.

$$\frac{9}{50} = \frac{9 \times 2}{50 \times 2} = \frac{18}{100}$$

Playing Video Games represents 18% of all after school time.

Reasonableness

Verify the fraction using the fact that Watching TV is $\frac{7}{100}$ more than Playing Video Games.

$$\frac{18}{100} + \frac{7}{100} = \frac{25}{100}$$

Since Playing Video Games plus $\frac{7}{100}$ is $\frac{25}{100}$ and this is the same percent fraction value as Watching TV, these sectors are correct.

The sector marked 115.2° is Sports Activities, the sector marked 90° is Watching TV, the sector marked 64.8° is Playing Video Games, the sector marked 54° is Homework, and the sector marked 36° is Other Activities.

Of all after school time, on average:

- 32% or $\frac{8}{25}$ of the time is spent in Sports Activities
- 25% or $\frac{1}{4}$ of the time is spent Watching TV
- 18% or $\frac{9}{50}$ of the time is spent Playing Video Games
- 15% or $\frac{3}{20}$ of the time is spent doing Homework
- 10% or $\frac{1}{10}$ of the time is spent on Other Activities

To find the portion of the 7 hours of average after school time for each activity, multiply the fraction or the decimal value for each sector times 7. To use the decimal value, divide each percent value by 100 to convert the percents to decimals.

- Sports Activities: 32% = 0.32 × 7 = 2.24 hours
- Watching TV: 25% = 0.25 × 7 = 1.75 hours
- Playing Video Games: 18% = 0.18 × 7 = 1.26 hours
- Homework: 15% = 0.15 × 7 = 1.05 hours
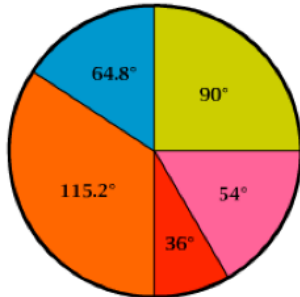- Other Activities: 10% = 0.1 × 7 = 0.7 hours

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Look For...

- a solution strategy to determine the activity related to each sector of the circle graph
- a solution strategy to determine the fraction of the circle graph composed by each activity
- a solution strategy to determine the percent of after school time used by each activity
- correct fraction and percent values for each sector of the graph relating to each activity
- correct time values in hours for each activity given an average of 7 hours after school time
- student justification of choices of solution strategy

Performing Math
open-middle performance tasks

Cluster 6.4: Proportionality

🔵 Differentiation: Simplified Task

Students in Study Skills class completed a survey about after school time by tracking the amount of time spent in different after school activities for a week. The teacher combined all the information from her students and produced the following graph.
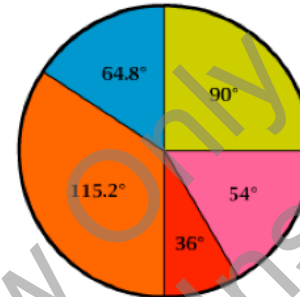

The *homework* section is 1.5 times larger than the *other activities* section. The largest amount of time after school is spent on *sports activities*. The *other activities* section is the smallest. The fraction of time spent *watching TV* is 0.07 more than the fraction of time spent *playing video games*. What fraction of all after school time is spent on each activity in the graph? What percent of after school time does each sector of the graph represent? Justify your reasoning.

Answer: The 115.2° sector is Sports Activities with $\frac{8}{25}$ or 32% of time. The 64.8° sector is Playing Video Games with $\frac{9}{50}$ or 18% of time. The 90° sector is Watching TV with $\frac{1}{4}$ or 25% of time. The 54° sector is Homework with $\frac{3}{20}$ or 15% of time. The 36° sector is Other Activities with $\frac{1}{10}$ or 10% of time.

🟦 Differentiation: Enriching Task

Students in Study Skills class completed a survey about after school time by tracking the amount of time spent in different after school activities for a week. The teacher combined all the information from her students and produced the following graph.

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Answer: The 115.2° sector is Sports Activities with $\frac{8}{25}$ or 32% of time. The 64.8° sector is Playing Video Games with $\frac{9}{50}$ or 18% of time. The 90° sector is Watching TV with $\frac{1}{4}$ or 25% of time. The 54° sector is Homework with $\frac{3}{20}$ or 15% of time. The 36° sector is Other Activities with $\frac{1}{10}$ or 10% of time. Of a 7-hour period of after school time, the average student would spend 2 hours 14 minutes 24 seconds in Sports Activities, 1 hour 45 minutes Watching TV, 1 hour 15 minutes 36 seconds Playing Video Games, 1 hour 3 minutes doing Homework, and 42 minutes doing Other Activities.

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Scaffolded Task with Answers

Students in Study Skills class completed a survey about after school time by tracking the amount of time spent in different after school activities for a week. The teacher combined all the information from her students and produced the following graph.
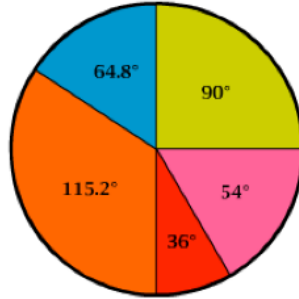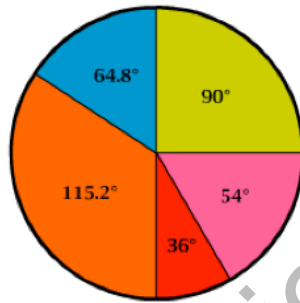

The *homework* section is 1.5 times larger than the *other activities* section. The largest amount of time after school is spent on *sports activities*. The fraction of time spent *watching TV* is 0.07 more than the fraction of time spent *playing video games*.

1. What activity relates to each sector?
   115.2° sector = Sports Activities; 64.8° sector = Playing Video Games; 90° sector = Watching TV; 54° sector = Homework; 36° sector = Other Activities

2. Use an appropriate strategy to determine the fraction of the degrees in the circle comprising each sector of the graph.
   Sports Activities = $\frac{8}{25}$; Playing Video Games = $\frac{9}{50}$; Watching TV = $\frac{1}{4}$; Homework = $\frac{3}{20}$; Other Activities = $\frac{1}{10}$

3. Convert the fraction for each sector to a percent.
   Sports Activities = 32%; Playing Video Games = 18%; Watching TV =25%; Homework =15%; Other Activities = 10%

4. What fraction of all after school time is spent on each activity in the graph?
   Sports Activities = $\frac{8}{25}$; Playing Video Games = $\frac{9}{50}$; Watching TV = $\frac{1}{4}$; Homework = $\frac{3}{20}$; Other Activities = $\frac{1}{10}$

5. What percent of after school time does each sector of the graph represent?
   Sports Activities = 32%; Playing Video Games = 18%; Watching TV = 25%; Homework =15%; Other Activities = 10%

6. If the average student has 7 hours of after school time before going to bed, what portion, in decimal form, of this time is spent on each activity?
   Sports Activities = 2.24 hours; Playing Video Games = 1.26 hours; Watching TV = 1.75 hours; Homework =1.05 hours; Other Activities = 0.7 hours

Name _____ Date _____

Students in Study Skills class completed a survey about after school time by tracking the amount of time spent in different after school activities for a week. The teacher combined all the information from her students and produced the following graph.

The *homework* section is 1.5 times larger than the *other activities* section. The largest amount of time after school is spent on *sports activities*. The fraction of time spent *watching TV* is 0.07 more than the fraction of time spent *playing video games*.

- What fraction of all after school time is spent on each activity in the graph?
- What percent of after school time does each sector of the graph represent?
- If the average student has 7 hours of after school time before going to bed, what portion of this time is spent on each activity?

Justify your reasoning.

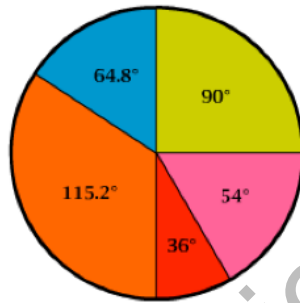|			
Procedural	0	1	2
Conceptual	0	1	2
Communication	0	1	2

Total points:_____

Performance Task: 6.4G
Rates and Ratios: After School Time

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- What fraction of all after school time is spent on each activity in the graph?
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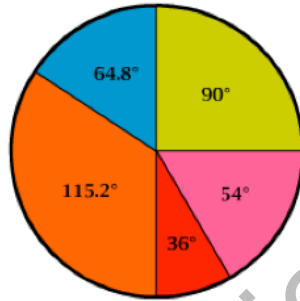
Justify your reasoning.

Procedural	0	1	2
Conceptual	0	1	2
Communication	0	1	2

Total points:_____

Name _____ Date _____

Students in Study Skills class completed a survey about after school time by tracking the amount of time spent in after school different activities for a week. The teacher combined all the information from her students and produced the following graph.

The *homework* section is 1.5 times larger than the *other activities* section. The largest amount of time after school is spent on *sports activities*. The fraction of time spent *watching TV* is 0.07 more than the fraction of time spent *playing video games*.

- What fraction of all after school time is spent on each activity in the graph?
- What percent of after school time does each sector of graph represent?
- If the average student has 7 hours of after school time before going to bed, what amount of time in hours, minutes, and seconds is spent on each activity?

Justify your reasoning.

Procedural	0	1	2
Conceptual	0	1	2
Communication	0	1	2

Total points:_____

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1. What activity relates to each sector?

2. Use an appropriate strategy to determine the fraction of the degrees in the circle comprising each sector of the graph.

3. Convert the fraction for each sector to a percent.

4. What fraction of all after school time is spent on each activity in the graph?

5. What percent of after school time does each sector of the graph represent?

6. If the average student has 7 hours of after school time before going to bed, what portion, in decimal form, of this time is spent on each activity?