SOLVING PROBLEMS WITH VOLUME

The student is expected to solve problems involving the volume of cylinders, cones, and spheres.

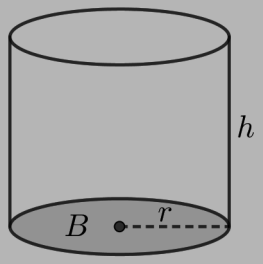
TELL ME MORE...

The **volume** of a 3-dimensional figure is the amount of space taken up by the interior of the figure.

- A **cylinder** is a three-dimensional figure with two congruent, parallel bases that are circles. The **height** of the cylinder is the distance between the parallel **bases**.
- A **cone** is a three-dimensional figure with one base that is a circle. The lateral surface of a cone narrows and comes to a point, called the **vertex** or **apex** of the cone, opposite the base. The **height** of the cone is the perpendicular distance between the vertex and the base.
- A **sphere** is a three-dimensional figure that is a set of all points in space that are equidistant from one point, called the **center** of the sphere. The radius of a sphere is the distance from the center of the sphere to the surface of the sphere.

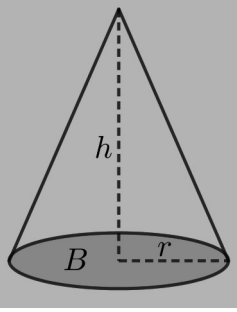
CYLINDER: $V = Bh$

- B represents the area of the base of the cylinder, which is a circle.
- $B = \pi r^2$, r represents the radius of the base.
- h represents the height of the cylinder.



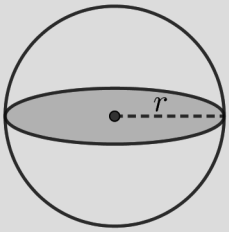
CONE: $V = \frac{1}{3}Bh$

- B represents the area of the base of the cone, which is a circle.
- $B = \pi r^2$, r represents the radius of the base.
- h represents the height of the cone.



SPHERE: $V = \frac{4}{3}\pi r^3$

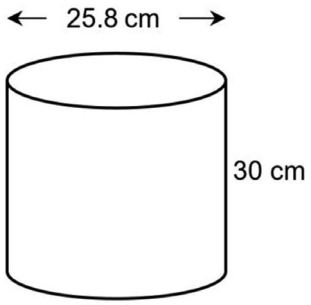
- r represents the radius of the sphere.



In any formula involving π, you may use an approximation of π such as 3.14, $\frac{22}{7}$, or the π key on your calculator.

EXAMPLES

EXAMPLE 1: Travis uses a container shaped like a cylinder to carry fish food to the pond to feed the fish. The dimensions of the container are shown. Determine the approximate volume of the container. Round to the nearest hundredth of a cubic centimeter.



STEP 1 The volume of a cylinder is found using the formula, $V = Bh$. The base of a cylinder is a circle, so use the formula for the area of a circle, $B = \pi r^2$. The area formula requires the radius of the base, so determine the radius of the base.

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- The diameter of the base of the cylinder is given in the diagram as 25.8 centimeters.
- The area formula for a circle requires the *radius*, r, of the circle.
- The radius of a circle is $\frac{1}{2}$ the diameter:

  $r = \frac{1}{2}d = \frac{1}{2}(25.8) = 12.9$ cm.

***r* = 12.9 centimeters**

STEP 2 Calculate the area of the base using the area formula, $B = \pi r^2$.

- Substitute $r = 12.9$ into the area formula.
- $B = \pi(12.9)^2 = 166.41\pi$ square centimeters

***B* = 166.41π square centimeters**

STEP 3 Determine the approximate volume of the cylinder using the volume formula, $V = Bh$.

- The height of the cylinder is given in the diagram as 30 centimeters.
- Substitute $B = 166.41\pi$ and $h = 30$ into the volume formula.
- $V = 166.41\pi(30) \approx 15{,}675.82$ cubic centimeters

***V* ≈ 15,675.82 cubic centimeters**

YOU TRY IT!

A water storage tank is shaped like a cylinder. The radius of the tank is $3\frac{1}{2}$ feet and the height of the tank is 5 feet. Determine the volume of the tank in cubic feet. Round your answer to the nearest hundredth.

Radius of tank: $r =$ _____

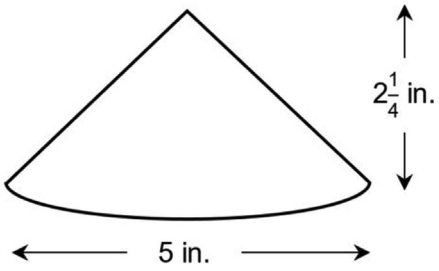
Area of base of tank:

$B = \pi($_____$)^2 =$ _____π
(r)

Volume:

$V =$ _____$\pi($_____$) \approx$ _____
(B) (h)

EXAMPLE 2: A cone has the dimensions shown in the diagram. Determine the approximate volume of the cone in cubic inches. Round your answer to the nearest hundredth.



STEP 1 The volume of a cone is found using the formula, $V = \frac{1}{3}Bh$. The base of a cone is a circle, so use the formula for the area of a circle, $B = \pi r^2$. The area formula requires the radius of the base, so determine the radius of the base.

- The diameter of the base of the cone is given in the diagram as 5 inches.
- The area formula for a circle requires the *radius*, r, of the circle.
- The radius of a circle is $\frac{1}{2}$ the diameter: $r = \frac{1}{2}d = \frac{1}{2}(5) = 2\frac{1}{2}$ inches.

***r* = $2\frac{1}{2}$ inches**

STEP 2 Calculate the area of the base using the area formula, $B = \pi r^2$.

Substitute $r = 2\frac{1}{2}$ into the area formula.

$B = \pi\left(2\frac{1}{2}\right)^2 = 6\frac{1}{4}\pi$ square inches

***B* = $6\frac{1}{4}$π square inches**

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STEP 3 Determine the approximate volume of the cone using the volume formula, $V = Bh$.

- The height of the cone is given in the diagram as $2\frac{1}{4}$ inches.
- Substitute $B = 6\frac{1}{4}\pi$ and $h = 2\frac{1}{4}$ into the volume formula.
- $V = 6\frac{1}{4}\pi\left(2\frac{1}{4}\right) \approx 44.16$ cubic inches

***V* ≈ 44.16 cubic inches**

EXAMPLE 3: A ball shaped like a sphere has a radius of 8.4 centimeters. Determine the approximate volume of the ball in cubic centimeters. Round your answer to the nearest hundredth.

STEP 1 The volume of a sphere is found using the formula, $V = \frac{4}{3}\pi r^3$. Use this formula to determine the approximate volume of the ball.

- The radius of the ball is given as 8.4 centimeters.
- Substitute $r = 8.4$ into the volume formula.
- $V = \frac{4}{3}\pi(8.4)^3 \approx 2481.45$ cubic centimeters

***V* ≈ 2481.45 cubic centimeters**

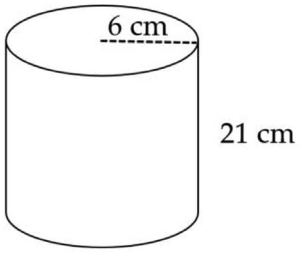
MAKE A NOTE . . .

Solve Example 3 using $\pi \approx 3.14$. Then, solve Example 3 using the π key on your calculator. Why are the two answers different?

PRACTICE

For questions 1-3, determine the approximate volume to the nearest hundredth.

1.



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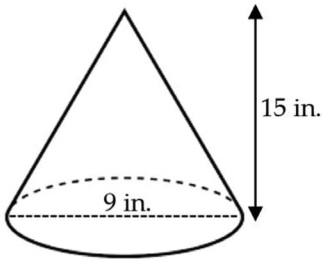
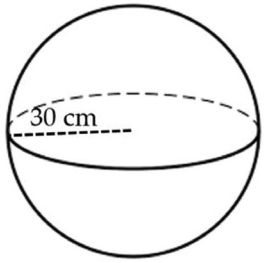


3.



4. A 12-foot section of water pipe is in the shape of a cylinder and is being installed by construction crews. The pipe section has a diameter of 8.5 feet. To the nearest hundredth, what is the approximate volume of the pipe section?

5. A baseball is in the shape of a sphere and has a diameter of 74 millimeters. What is the approximate volume of the baseball, to the nearest hundredth?

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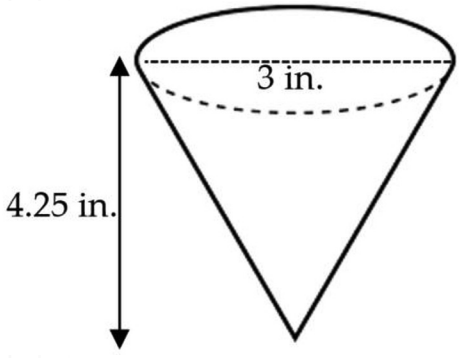
6. Andrea is excited to order a new cookie cone filled with ice cream at an ice cream shop. The cookie cone has a height of 15 centimeters and a radius of 5.5 centimeters. What is the approximate volume, to the nearest hundredth, of the cone?

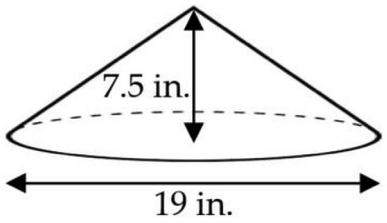
7. Romero is using a cylindrical glass beaker for a science experiment. The beaker is 13 centimeters tall and has a radius of 5 centimeters. Romero fills the beaker half way with a liquid. What is the approximate volume, to the nearest hundredth, of the liquid inside the beaker?

8. The paper drinking cups in Mr. Maldonado's office building are in the shape of a cone with dimensions as shown below.



If Mr. Maldonado fills a drinking cone $\frac{3}{4}$ of the way full with water, to the nearest tenth what is the approximate volume of water in the drinking cone?

9. Lui has a hat passed down from his grandfather that he wears as part of a family custom for certain ceremonies. The hat is in the shape of a cone with a diameter of 19 inches and a height from the center of 7.5 inches.



Which is closest to the volume of the conical hat in cubic inches?

A 2,834 $in.^3$

B 2,125 $in.^3$

C 708 $in.^3$

D 75 $in.^3$

10. In the downtown area of a town stands an old grain silo in the shape of a cylinder. The silo is 25 feet tall and has a diameter of 12.4 feet. Which measurement is closest to the volume of the grain silo in town in cubic feet?

F 12,070.2 ft^3

G 3,017.5 ft^3

H 973.4 ft^3

J 310 ft^3

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