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Using Linear and Absolute Value Functions

Explore – Answer Key

Directions: Use the information in the problem to complete the table and answer the questions.

Part 1: Formulating Absolute Value Functions

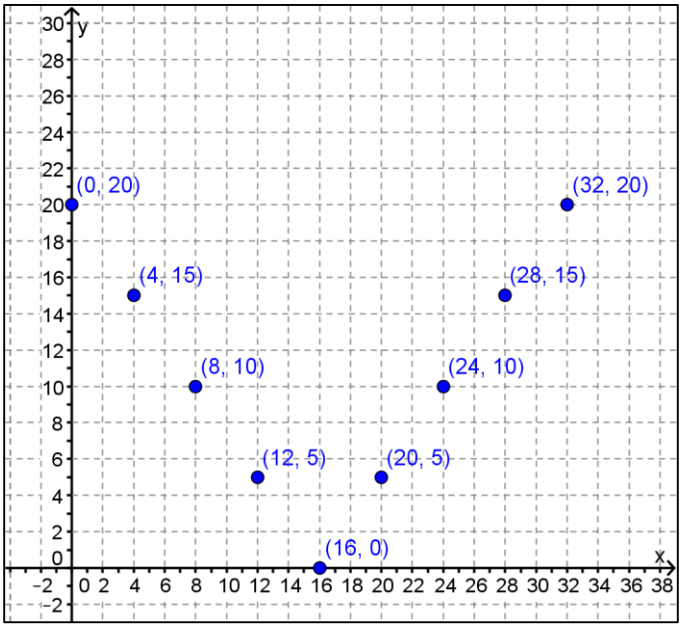
Francisco and his family were traveling to visit relatives. Along their drive, Francisco knew they would drive past the Cadillac Ranch. Francisco noticed that his father was driving an average speed of 75 miles per hour. He recorded the time and the distance they were from the Cadillac Ranch as they approached and then passed it. Francisco began recording the data when there were 20 miles until his family reached the Cadillac Ranch.

Cadillac Ranch, Richie Diesterhe, Wikimedia Commons

1. Use the information in the problem to complete the table.

Time, x (minutes)	Distance, y (miles)
0	***20***
4	***15***
8	***10***
12	***5***
16	***0***
20	***5***
24	***10***
28	***15***
32	***20***

2. Make a graph of the data. Describe the shape of the graph, including rates of change and key points that affect the shape of the graph.
The data appear to be V-shaped. There is a constant rate of change of $-\frac{5}{4}$ until (16, 0), when the constant rate of change becomes $+\frac{5}{4}$. It appears as though two lines that intersect at (16, 0) will model the data.



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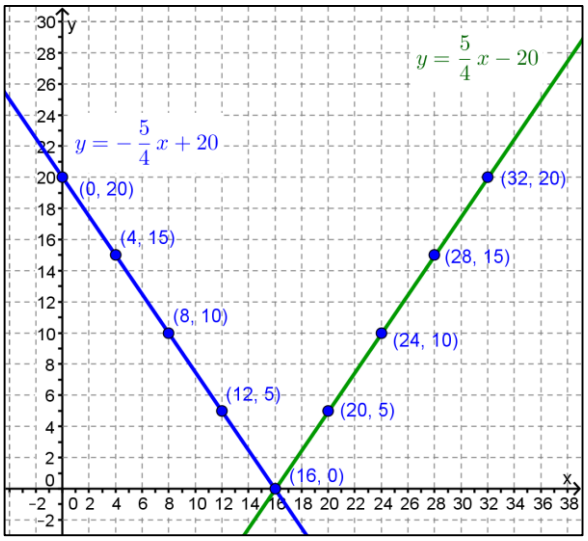
3. Use the graph and table to divide the data set into two different parts that could each be modeled with a different function. Write the functions and graph them over your data set.



Time, x (minutes)	Distance, y (miles)
0	20
4	15
8	10
12	5
16	0
20	5
24	10
28	15
32	20

This portion of the table has points with a constant rate of change, $-\frac{5}{4}$, and a y-intercept of (0, 20).
$f(x) = -\frac{5}{4}x + 20$

This portion of the table has points with a constant rate of change, $\frac{5}{4}$, and a y-intercept of (0, −20).
$f(x) = \frac{5}{4}x - 20$

4. For your set of functions, identify the domain for which your function models the data.
For $f(x) = -\frac{5}{4}x + 20$, the domain is [0, 16] and for $f(x) = \frac{5}{4}x - 20$, the domain is [16, 32].

5. Factor out −1 from all terms of the second function you wrote. How does the new function rule compare to the function rule you wrote for the first function?
The function $f(x) = \frac{5}{4}x - 20 = -(-\frac{5}{4}x + 20)$, which is the same as the first function with $a = -1$.

6. What transformation does $a = -1$ represent?
$a = -1$ represents a vertical reflection across the x-axis.

7. What do the following pairs of numbers have in common: $|6|$ and $|-6|$, $|-5.2|$ and $|5.2|$, and $|3x|$ and $|-3x|$?
Each pair contains two equivalent numbers. $|6| = |-6| = 6$, $|-5.2| = |5.2| = 5.2$, and $|3x| = |-(3x)| = 3x$.

8. How could you write your set of functions as one function?
$f(x) = |-\frac{5}{4}x + 20|$

9. Rewrite your function by factoring out −1 and then taking its absolute value to generate the value of a.
$f(x) = |\frac{5}{4}x - 20|$

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Part 1: Debriefing Questions

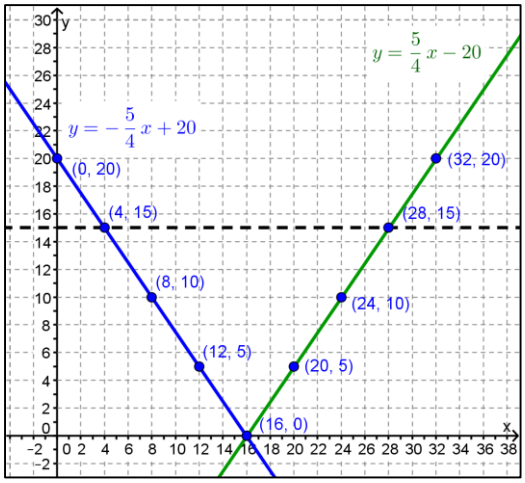
1. How are the two functions you wrote from the graph and table related?
   The two functions have slopes that have opposite signs and have y-intercepts whose y-coordinates have opposite signs.
   One function is a vertical reflection of the other function across the x-axis.

2. The **absolute value** of a number is its distance from 0 along a number line. In this situation, what point behaves as the pivot point, like 0 on a number line, in which the distance from the point stops decreasing and begins increasing?
   The x-intercept of the graph, (16, 0), serves as a pivot point.

Part 2: Solving Absolute Value Equations

1. Suppose that Fernando is 15 miles away from the Cadillac Ranch. Use your absolute value function from Part 1 to write an equation that Fernando could solve to determine the time the family has been driving since he began recording data.
   $15 = \left|\frac{5}{4}x - 20\right|$

2. Use your graph to estimate the solution to your equation.
   $x = 4, 28$



3. Use the two functions you originally wrote from your graph in Part 1 to write two equations where the *y*-value is 15.
   $15 = -\frac{5}{4}x + 20$ $\qquad$ $15 = \frac{5}{4}x - 20$

4. Solve each of these equations for *x* using inverse operations.

$15 = -\frac{5}{4}x + 20$	$15 = \frac{5}{4}x - 20$
$15 - 20 = -\frac{5}{4}x + 20 - 20$	$15 + 20 = \frac{5}{4}x - 20 + 20$
$-5 = -\frac{5}{4}x$	$35 = \frac{5}{4}x$
$-\frac{4}{5}(-5) = -\frac{4}{5}(-\frac{5}{4}x)$	$\frac{4}{5}(35) = \frac{4}{5}(\frac{5}{4}x)$
$4 = x$	$28 = x$

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5. If $|x| = 3$, then $x = 3$ or $x = -3$. If $|2x| = 5$, then $2x = 5$ or $-(2x) = 5$. Use this pattern and your absolute value function from Question 1 to write two equations for y.

   $y = \frac{5}{4}x - 20$ $\qquad$ $y = -\left(\frac{5}{4}x - 20\right)$

6. For Fernando's problem, he wants to know at what times he and his family were 15 miles away from the Cadillac Ranch. For both of the equations you just wrote, let $y = 15$. Solve each equation for x.

   $15 = \frac{5}{4}x - 20$
   $15 + 20 = \frac{5}{4}x - 20 + 20$
   $35 = \frac{5}{4}x$
   $\frac{4}{5}(35) = \frac{4}{5}(\frac{5}{4}x)$
   $28 = x$

   $15 = -\left(\frac{5}{4}x - 20\right)$
   $15 \div -1 = -\left(\frac{5}{4}x - 20\right) \div -1$
   $-15 = \frac{5}{4}x - 20$
   $-15 + 20 = \frac{5}{4}x - 20 + 20$
   $5 = \frac{5}{4}x$
   $4 = x$

Part 2: Debriefing Questions

1. How do the solutions for the two linear equations compare to the solutions for the absolute value equation you obtained from the graph?
   The solutions are the same.

2. How can you solve absolute value equations with one variable using inverse operations?
   Write two equations: one that is the positive quantity inside the absolute value symbol and one that is the negative quantity inside the absolute value symbol. Solve each equation.