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## Using Linear and Absolute Value Functions

Elaborate - Answer Key

Directions: The absolute value parent function is $y=|x|$. The parameters $a, b, c$, and $d$, which represent real numbers can be used to transform the absolute value parent function: $y=a|x|, y=|b x|, y=|x-c|$, and $y=|x|+d$. Use your graphing calculator to graph the four functions shown in each box on the same screen. Graph the first function, $Y 1$, in bold or a different color. Use the graphs and tables of values on the graphing calculator to answer the questions next to the box.

Part 1: Investigating a

- $Y 1=|x|$
- $Y 2=2|x|$
- $Y 3=4|x|$
- $Y 4=5|x|$
- $Y 1=|x|$
- $Y 2=0.5|x|$
- $Y 3=0.25|x|$
- $Y 4=0.1|x|$

2. What happens to the graph of $y=a|x|$ when the value of $a$ decreases?
As a decreases, the graph becomes more vertically compressed because the $y$-values are moved closer to the $x$-axis.

|  | X | $\mathrm{Y}_{1}$ | $\mathrm{Y}_{2}$ | Y 3 | Y 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| " " - " " | -5 | 5 | 2.5 | 1.25 | . 5 |
| - | -4 | 4 | 2 | 1 | . 4 |
| - | -3 | 3 | 1.5 | . 75 | . 3 |
|  | -2 | 2 | 1 | . 5 | . 2 |
| $\rightarrow 1 \leq$ | -1 | 1 | . 5 | . 25 | . 1 |
|  | 0 | 0 | 0 | 0 | 0 |
| " " " " " | 1 | 1 | . 5 | . 25 | . 1 |
| . . . . . . - . . . . . | 2 | 2 | 1 | . 5 | . 2 |
| $\ldots$. $+\ldots \ldots$ | 3 | 3 | 1.5 | . 75 | . 3 |
| " " * " * * " " . | 4 | 4 | 2 | 1 | . 4 |
|  | 5 | 5 | 2.5 | 1.25 | . 5 |

- $Y 1=|x|$
- $Y 2=-|x|$
- $Y 3=3|x|$
- $Y 4=-3|x|$

3. What happens to the graph of $y=a|x|$ when the value of $a$ changes signs from positive to negative?
When a changes signs, the graph is reflected across the x-axis.

| \†7" | X | Y 1 | Y2 | $\mathrm{Y}_{3}$ | Y4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | -5 | 5 | -5 | 15 | -15 |
| , | -4 | 4 | -4 | 12 | -12 |
| , | -3 -2 | 3 | -3 | 9 | -9 |
| , | -2 | 2 | -2 | 6 | -6 |
| 1 | ${ }^{-1}$ |  | ${ }^{-1}$ | 3 |  |
| † | 1 | 1 | -1 | 3 | -3 |
| $\ldots .$. | 2 | 2 | -2 | 6 | -6 |
| $\square>$ | 3 | 3 | -3 | 9 | -9 |
|  | 4 5 | 4 5 | -4 -5 | 12 | -12 |

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- $\quad Y 1=|x|+1$
- $Y 2=-|x|+1$
- $\quad Y 3=3|x|+1$
- $Y 4=-3|x|+1$

4. What happens to the graph of $y=a|x|+1$ when the value of a changes signs from positive to negative?
When a changes signs, the graph is reflected across the line $y=1$.

|  | X | Y 1 | $\mathrm{Y}_{2}$ | $Y_{3}$ | $Y_{4}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| ' -1 | -5 | 6 | -4 | 16 | -14 |
| - | -4 | 5 | -3 | 13 | -11 |
| N | -3 | 4 | -2 | 10 | -8 |
| * | -2 | 3 | -1 | 7 | -5 |
| 1 , | -1 | 2 | 0 | 4 | -2 |
| - $/$ | 0 | 1 | 1 | 1 | 1 |
| - - | 1 | 2 | 0 | 4 | -2 |
| - | 2 | 3 | -1 | 7 | -5 |
| - | 3 | 4 | -2 | 10 | -8 |
| " ${ }^{\prime \prime} \cdot{ }^{\prime}$ | 4 | 5 | -3 -4 | 13 | -11 |
| , 1.1 .1 | 5 | 6 | -4 | 16 | -14 |

5. What happens to the graph of $y=|b x|$ when the value of $b$ increases?
As $b$ increases, the graph becomes more horizontally compressed because the $x$-values are moved closer to the $y$-axis.


- $\quad Y 1=|x|$
- $Y 2=|0.5 x|$
- $Y 3=|0.25 x|$
- $Y 4=|0.1 x|$
- $\quad Y 1=|x-3|$
- $\quad Y 2=|-(x-3)|$
- $Y 3=|4(x-3)|$
- $\quad Y 4=|-4(x-3)|$

6. What happens to the graph of $y=|b x|$ when the value of $b$ decreases?
As b decreases, the graph becomes more horizontally stretched because the $x$-values are moved farther from the $y$-axis.

| - \ † . | X | Y 1 | $\mathrm{Y}_{2}$ | $\mathrm{Y}_{3}$ | $Y_{4}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | -5 | 5 | 2.5 | 1.25 | . 5 |
|  | -4 | 4 | 2 | 1 | . 4 |
|  | -3 | 3 | 1.5 | . 75 |  |
| - | -2 -1 | 2 | 1 <br> . | .5 .25 | .2 .1 |
| +1, | 0 | $\theta$ | $\stackrel{\square}{0}$ | 9 | - |
| - . . . . 7 . | 1 | 1 | . 5 | . 25 | . 1 |
| * . . . . . . . . . | 2 | 2 | 15 | . 75 | . 2 |
| - . . . . . . . . | 3 4 | 3 4 | ${ }_{2}^{1.5}$ |  | . ${ }^{\text {. }}$ |
| ..... | 5 | 5 | 2.5 | 1.25 | 5 |

7. What happens to the graph of $y=|b(x-3)|$ when the value of $b$ changes signs from positive to negative?
When $b$ changes signs, the graph is reflected across the line $y=c$. However, the pattern is not noticeable in the graph because of the symmetry of the graph across the line $y=c$.

| \} | X | Y 1 | Y2 | $Y_{3}$ | Y4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | -5 | 8 | 8 | 32 | 32 |
| - | -4 | ? | ? | 28 | 28 |
| - | -3 -2 | 6 5 | 6 5 | 24 | 24 |
| , | -1 | 4 | 4 | 16 | 16 |
|  | ${ }^{0}$ | 3 | 3 | 12 | 12 |
| " " . " . $\quad$. " " . | 1 | 2 | 2 | 8 | 8 |
| - . . . $\quad$. . . | $\frac{2}{3}$ | $\frac{1}{0}$ | 1 | 0 | 4 |
| . . . . . . . . . . | 4 | 1 | 1 |  | - |
|  | 5 | 2 | 2 |  |  |

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Part 3: Investigating c

- $Y 1=|x|$
- $Y 2=|x-2|$
- $Y 3=|x-3|$
- $Y 4=|x-5|$
- $Y 1=|x|$
- $Y 2=|x+1|$
- $Y 3=|x+3|$
- $Y 4=|x+5|$


## Part 4: Investigating d

- $Y 1=|x|$
- $Y 2=|x|+1$
- $Y 3=|x|+2$
- $Y 4=|x|+3$
- $Y 1=|x|$
- $Y 2=|x|-1$
- $Y 3=|x|-2$
- $Y 4=|x|-3$

9. What happens to the graph of $y=|x-c|$ when the value of $c$ is negative and decreases?
As c decreases, the graph shifts or translates cunits to the left of the parent function.

| - 7 / | X | Y 1 | Y 2 | Y 3 | Y4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| - | -5 | 5 | 4 | 2 | 0 |
| - | -4 | 4 | 3 | 1 | 1 |
| $\cdots$ | -3 | 3 | 2 | 0 | 2 |
| $\cdots$ | -2 | 2 | 1 | 1 | 3 |
| 1, | -1 | 1 | 0 | 2 | 4 |
|  | 0 | 0 | 1 | 3 | 5 |
| " " " . " " f " . . . . | 1 | 1 | 2 | 4 | 6 |
| " - " . . $\dagger$ - " . . | 2 | 2 | 3 | 5 | ? |
| . . . . . . . . . . . | 3 | 3 | 4 | 6 | 8 |
| " " " . " f " . . . . | 4 | 4 5 | 5 6 | ? | ${ }^{9} 10$ |
|  | 5 | 5 | 6 | 8 | 10 |

8. What happens to the graph of $y=|x-c|$ when the value of c increases?
As cincreases, the graph shifts or translates c units to the right from the parent function.


| $X$ | $Y_{1}$ | $Y_{2}$ | $Y_{3}$ | $Y_{4}$ |
| :--- | :--- | :--- | :--- | :--- |
| -5 | 5 | 7 | 8 | 10 |
| -4 | 4 | 6 | 7 | 9 |
| -3 | 3 | 5 | 6 | 8 |
| -2 | 2 | 4 | 5 | 7 |
| -1 | 1 | 3 | 4 | 6 |
| 0 | 0 | 2 | 3 | 5 |
| 1 | 1 | 1 | 2 | 4 |
| 2 | 2 | 0 | 1 | 3 |
| 3 | 3 | 1 | 0 | 2 |
| 4 | 4 | 2 | 1 | 1 |
| 5 | 5 | 3 | 2 | 0 |


10. What happens to the graph of $y=|x|+d$ when the value of $d$ increases?
As $d$ increases, the graph shifts or translates $d$ units upward from the original function.

|  | $X$ | $Y_{1}$ | $Y_{2}$ | $Y_{3}$ | $Y_{4}$ |
| :---: | :---: | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |

11. What happens to the graph of $y=|x|+d$ when the value of $d$ is negative and decreases?
As d decreases, the graph shifts or translates d units downward from the original function.


| $X$ | $Y_{1}$ | $Y_{2}$ | $Y_{3}$ | $Y_{4}$ |
| :--- | :--- | :--- | :--- | :--- |
| -5 | 5 | 4 | 3 | 2 |
| -4 | 4 | 3 | 2 | 1 |
| -3 | 3 | 2 | 1 | 0 |
| -2 | 2 | 1 | 0 | -1 |
| -1 | 1 | 0 | -1 | -2 |
| 0 | 0 | -1 | -2 | -3 |
| 1 | 1 | 0 | -1 | -2 |
| 2 | 2 | 1 | 0 | -1 |
| 3 | 3 | 2 | 1 | 0 |
| 4 | 4 | 3 | 2 | 1 |
| 5 | 5 | 4 | 3 | 2 |

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## Debriefing Questions

1. In general, how does the parameter a affect the graph of $f(x)$ when it is changed to $a \cdot f(x)$ ? The parameter a generates a vertical dilation or reflection. If a>1, then the graph of $f(x)$ is vertically dilated by a factor of $a$. If $0<a<1$, then the graph of $f(x)$ is vertically compressed by a factor of $a$. If $a<1$, then the graph of $f(x)$ is reflected across the horizontal line containing the vertex of the absolute value graph.
2. In general, how does the parameter $b$ affect the graph of $f(x)$ when it is changed to $f(b x)$ ? The parameter $\boldsymbol{b}$ generates a horizontal dilation or reflection. If $\boldsymbol{b}>1$, then the graph of $f(x)$ is horizontally compressed by a factor of $\frac{1}{b}$. If $0<b<1$, then the graph of $f(x)$ is horizontally stretched by a factor of $\frac{1}{b}$. If $b<1$, then the graph of $f(x)$ is reflected across the vertical line containing the vertex of the absolute value graph.
3. In general, how does the parameter $c$ affect the graph of $f(x)$ when it is changed to $f(x-c)$ ? The parameter c generates a horizontal translation. If $\mathrm{c}>0$, then the graph is translated c units to the right. If $\mathrm{c}<0$, then the graph is translated c units to the left.
4. In general, how does the parameter $d$ affect the graph of $f(x)$ when it is changed to $f(x)+d$ ? The parameter $d$ generates a vertical translation. If $d>0$, then the graph is translated $d$ units up. If $\mathbf{d}<0$, then the graph is translated $d$ units down.
