Cluster G.2: Coordinate and Transformational Geometry

G.2C: Parallel and Perpendicular Lines: Four Towns

Focusing TEKS

G.2C Coordinate and transformational geometry. The student uses the process skills to understand the connections between algebra and geometry and uses the one- and two-dimensional coordinate systems to verify geometric conjectures. The student is expected to determine an equation of a line parallel or perpendicular to a given line that passes through a given point.

Additional TEKS:

A.2E Write the equation of a line that contains a given point and is parallel to a given line. **Supporting Standard**

A.2F Write the equation of a line that contains a given point and is perpendicular to a given line. **Supporting Standard**

Focusing Mathematical Process

G.1A Apply mathematics to problems arising in everyday life, society, and the workplace.

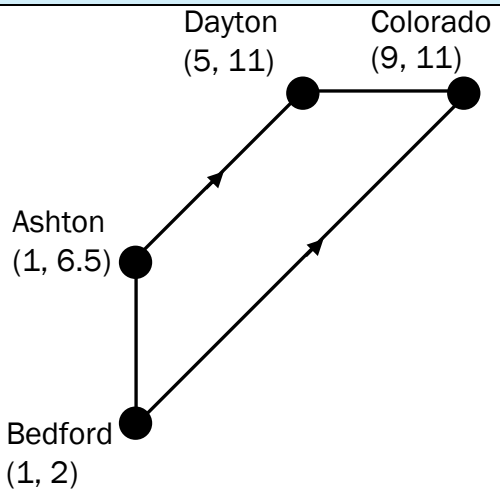
G.1B Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution.

G.1F Analyze mathematical relationships to connect and communicate mathematical ideas.

Performance Task

Four towns can be connected with roads that create a trapezoid. The center of each town can be represented on a map with a point on a coordinate plane as shown in the figure. A utility line will be run along the line containing the height of the trapezoid that passes through Dayton. What is the equation of this line?

Justify your reasoning.



Answer: $y = -\frac{8}{9}x + 15\frac{4}{9}$ or $8x + 9y = 139$

Mathematically Speaking...

In this task, students will write an equation of a line that passes through a given point (the location of Dayton) and is perpendicular to a given line (the segment connecting Bedford and Colorado). Students know that the line is perpendicular because it is the height of the trapezoid and the segment connecting Bedford and Colorado is one of the two parallel bases of the trapezoid.

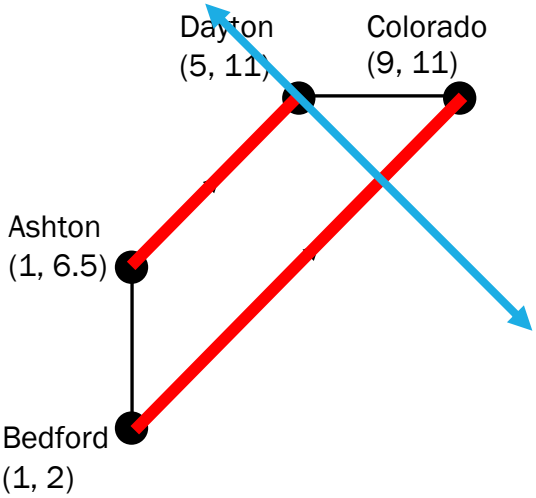
There are several viable solution pathways to this problem.

- Determine the equation of the line containing the segment between Bedford and Colorado using the point-slope formula, use the relationship between slopes of perpendicular lines to determine the slope of the line containing the height of the trapezoid, then use the point-slope formula with the location of Dayton as the point to write the equation of the line containing the height.
- Use the segment connecting Ashton and Bedford as the base of the trapezoid.
- Determine the slope of one of the parallel bases, use the relationship between slopes of perpendicular lines to determine the slope of the height, and use the coordinates of Dayton and slope-intercept form of a line to solve for *b*.
- Use dynamic geometry software to plot the four vertices of the trapezoid, draw the height through Dayton, and calculate the equation of that line.

Possible Solution

Analyzing the given information

In the diagram, there are two parallel lines marked with arrows. These two lines are the parallel bases of the trapezoid. The height of the trapezoid is a line segment that is perpendicular to one of these two bases. The line containing this segment is where the utility line will be placed. The question asks for the equation of this line.



Formulating a plan

- Determine the slope of the line segment connecting Bedford and Colorado.
- Use the relationship between slopes of perpendicular lines to determine the slope of the height of the trapezoid.
- Write the equation of the line containing the height of the trapezoid using that slope and the coordinates of Dayton.

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Determining a solution

Use the slope formula to determine the slope of the line segment connecting Bedford and Colorado, which is one of the two parallel bases of the trapezoid.

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11 - 2}{9 - 1} = \frac{9}{8}$$

The height of the trapezoid is a segment perpendicular to both of the parallel bases. Perpendicular lines have slopes that are negative reciprocals. Any line containing the height of the trapezoid will be perpendicular to the line segment connecting Bedford and Colorado and have a slope that is the negative reciprocal of $\frac{9}{8}$.

$$\frac{9}{8} \rightarrow -\frac{8}{9}$$

The line containing the height of the trapezoid that passes through Dayton will have a slope of $-\frac{8}{9}$ and pass through the point (5, 11). Use the point-slope formula to write the equation of this line.

$$y - y_1 = m(x - x_1)$$

$$y - (11) = -\frac{8}{9}(x - 5)$$

$$9(y - 11) = -8(x - 5)$$

$$9y - 99 = -8x + 40$$

$$8x + 9y = 139$$

Look For...

- a correctly derived slope of the line segment connecting Bedford and Colorado
- understanding that the height of a trapezoid is perpendicular to either of the two parallel bases.
- understanding that the slopes of perpendicular lines are negative reciprocals or have a product that is equal to −1.
- correct use of either the slope-intercept form or the point-slope formula
- if the student chose to use technology, an explanation of how the student knows the two lines are perpendicular
- student justification of choices of solution strategy

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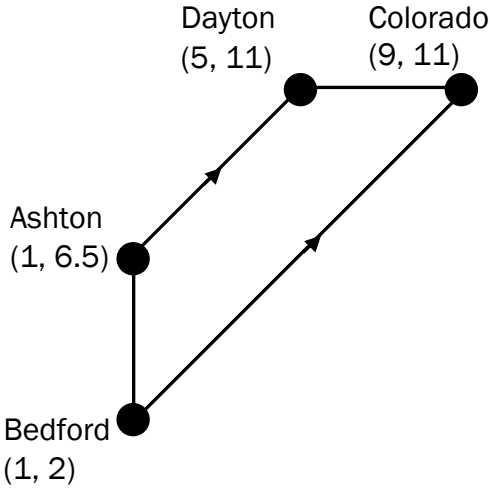
Differentiation: Simplified Task

Four towns can be connected with roads that create a trapezoid. The center of each town can be represented on a map with a point on a coordinate plane as shown in the figure. A utility line will be run from Dayton and will be perpendicular to the road connecting Bedford and Colorado. What is the equation of this line?



Justify your reasoning.

Answer:

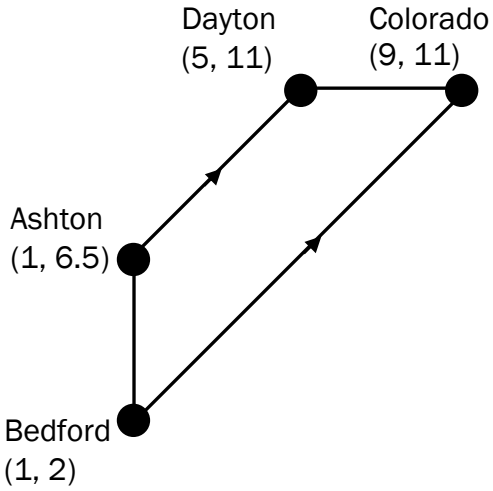
$y = -\frac{8}{9}x + 15\frac{4}{9}$ or $8x + 9y = 139$

Differentiation: Enriching Task

Four towns can be connected with roads that create a trapezoid. The center of each town can be represented on a map with a point on a coordinate plane as shown in the figure.

- A utility line will be run from Dayton and will be perpendicular to the road connecting Bedford and Colorado.
- A road will be constructed from Ashton directly to Colorado.

What are the approximate coordinates of the point where the utility line crosses the new road from Ashton to Colorado?



Justify your reasoning.

Answer: (13.87, 3.11)

The equation of utility line is $8x + 9y = 139$. The equation of the new road is $9x - 16y = 75$. Solving the system of these two linear equations generates a point of approximately (13.87, 3.11).

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Scaffolded Task with Answers

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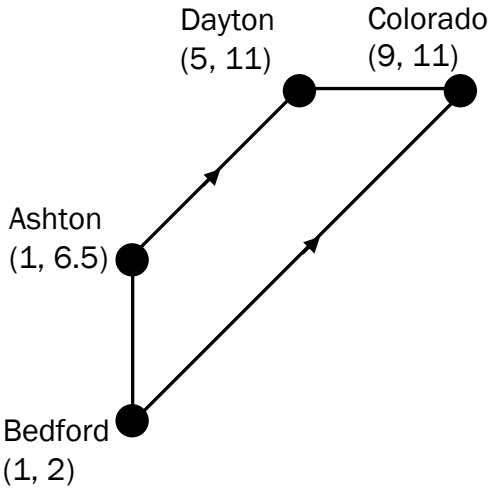

1. What is the slope of the road connecting Bedford and Colorado?
   $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11 - 2}{9 - 1} = \frac{9}{8}$

2. What part of the trapezoid is the road connecting Bedford and Colorado
   The road connecting Bedford and Colorado is one of the bases of the trapezoid.

3. What is the relationship between the road connecting Bedford and Colorado and the height of the trapezoid?
   The two segments are perpendicular.

4. What is the relationship between the slopes of two perpendicular lines?
   The slopes of two perpendicular lines are negative reciprocals of each other.

5. Determine the slope of the height of the trapezoid.
   $\frac{9}{8} \rightarrow -\frac{8}{9}$

6. What are the coordinates of Dayton on the map?
   (5, 11)

7. Use the slope of the height and the coordinates of Dayton to write the equation of the height of the trapezoid that passes through Dayton.
   $y - y_1 = m(x - x_1)$
   $y - (11) = -\frac{8}{9}(x - 5)$
   $9(y - 11) = -8(x - 5)$
   $9y - 99 = -8x + 40$
   $8x + 9y = 139$

Name _____ Date _____

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Parallel and Perpendicular Lines: Four Towns

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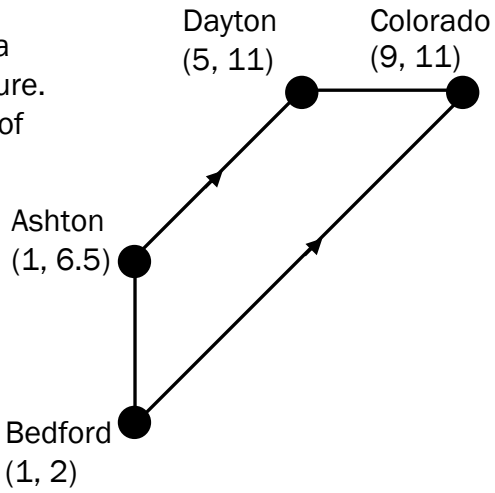

Justify your reasoning.

Procedural	0	1	2
Conceptual	0	1	2
Communication	0	1	2

Total points: _____

Name _____ Date _____

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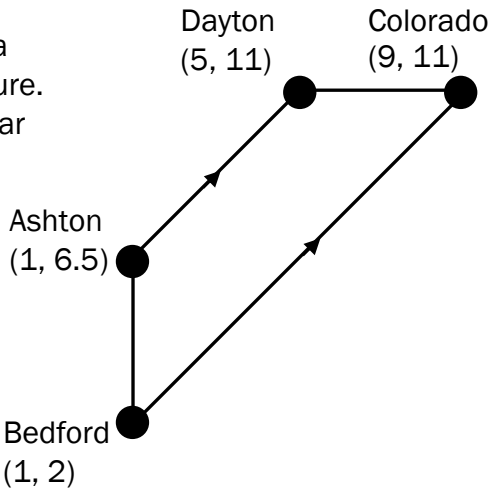
Justify your reasoning.



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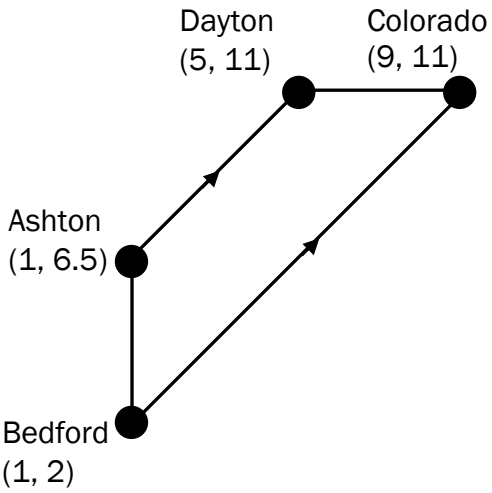

What are the approximate coordinates of the point where the utility line crosses the new road from Ashton to Colorado?

Justify your reasoning.

Procedural	0	1	2
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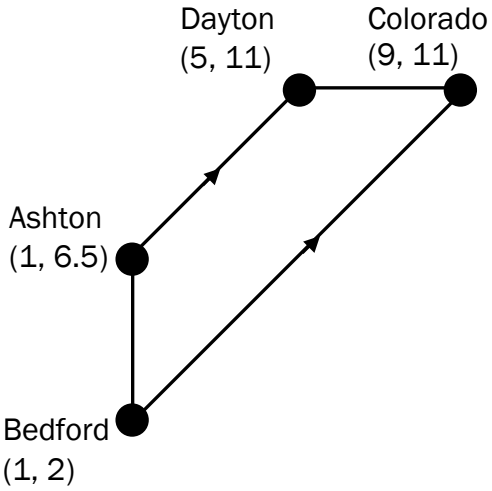

Justify your reasoning.

1. What is the slope of the road connecting Bedford and Colorado?

2. What part of the trapezoid is the road connecting Bedford and Colorado

3. What is the relationship between the road connecting Bedford and Colorado and the height of the trapezoid?

4. What is the relationship between the slopes of two perpendicular lines?

5. Determine the slope of the height of the trapezoid.

6. What are the coordinates of Dayton on the map?

7. Use the slope of the height and the coordinates of Dayton to write the equation of the height of the trapezoid that passes through Dayton.