Cluster G.12: Circles

G.12C: Area of Sectors of Circles: On the Radio

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Focusing TEKS

G.12C Circles. The student uses the process skills to understand geometric relationships and apply theorems and equations about circles. The student is expected to apply the proportional relationship between the measure of the area of a sector of a circle and the area of the circle to solve problems.

Additional TEKS:

G.11B Determine the area of composite two-dimensional figures comprised of a combination of triangles, parallelograms, trapezoids, kites, regular polygons, or sectors of circles to solve problems using appropriate units of measure.

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Focusing Mathematical Process

G.1A Apply mathematics to problems arising in everyday life, society, and the workplace.

G.1B Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution.

G.1E Create and use representations to organize, record, and communicate mathematical ideas

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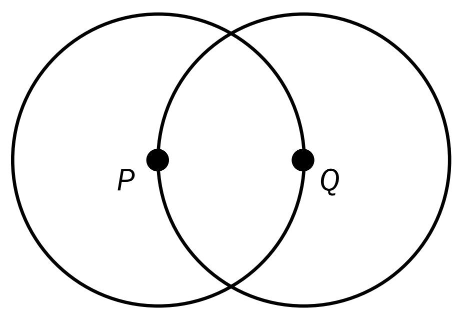
Performance Task

Two radio stations broadcast with the same intensity from two neighboring cities that are 90 miles apart. The signal from each station covers an area shaped like a circle. The signal is just strong enough to reach the transmitter from the other station so that the broadcast areas are congruent. What is the approximate area of the region that can listen to both radio stations?



Justify your reasoning.

Answer: *approximately 9950 square miles*

Cluster G.12: Circles

Mathematically Speaking...

In this task, students will use properties of a circle to decompose a region into smaller pieces for which area formulas are known.

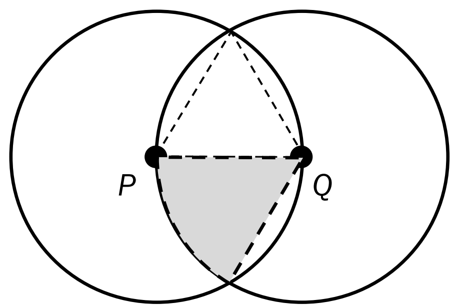
There are several viable solution pathways to this problem.

- Decompose the overlap region into a rhombus and four segments of circles with central angles of $60°$. Determine the area of each and add.
- Decompose the overlap region into one segment from each circle with a central angle of $120°$.
- Use a variety of methods to calculate the area of the triangle created by the sector and its corresponding segment, including properties of special right triangles or trigonometry to calculate the height of the triangle.

Possible Solution

Analyzing the given information

The region that receives a signal from both radio stations is the overlap between the two circles. Determine the area of this region, which is a combination of sectors of the circle, equilateral triangles, and segments of the circle.

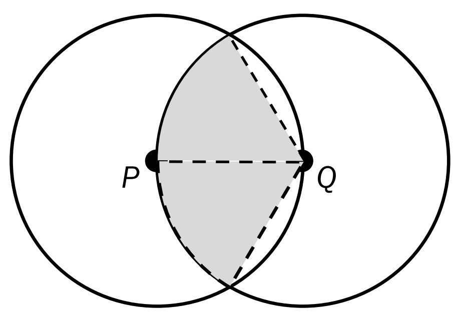


Formulating a plan

- Decompose the overlap region into figures for which there are known area formulas: sectors of a circle, equilateral triangles, or segments of a circle.
- Determine the areas of these non-overlapping figures.
- Calculate the total area by adding the areas of these non-overlapping figures.

Determining a solution

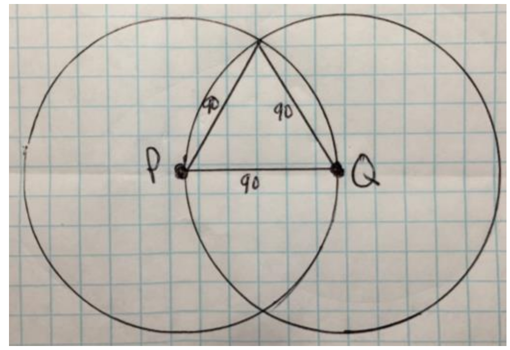
Decompose the overlap region into two sectors of circle Q. Determine the area of these sectors.



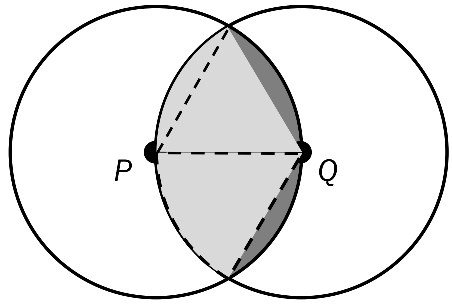
- The area of a sector is found using the formula $A = \frac{n}{360}(\pi r^2)$, where n is the measure of the central angle creating the sector and r is the radius of the circle.
- The radius of circle P is 90 miles and the radius of circle Q is also 90 miles. All radii of both circles are 90 miles, so any segment drawn from the center of circle P to its edge or circle Q to its edge is 90 miles.

- Drawing three radii as shown in the overlap region yields an equilateral triangle, so the central angle at either circle *P* or circle *Q* for this sector is 60°.

$$A_{\text{sector}} = \frac{n}{360}(\pi r^2)$$
$$A_{\text{sector}} = \frac{60}{360}(\pi(90)^2)$$
$$A_{\text{sector}} = \frac{1}{6}(8100)\pi$$
$$A_{\text{sector}} = 1350\pi \text{ square miles}$$



Notice that there are two regions left in the overlap region. Each of these regions is a segment of the other circle.

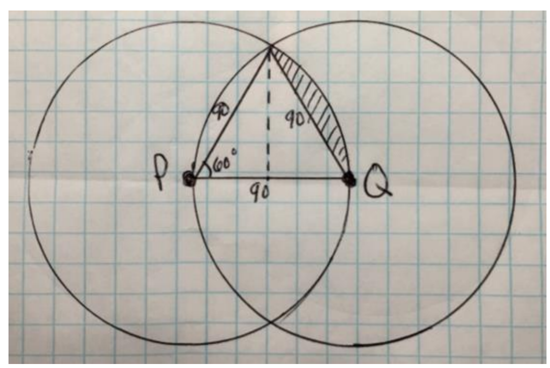


- The area of one segment of circle *P* is the area of the sector with the area of the equilateral triangle inside the sector removed.
- The area of an equilateral triangle can be determine using the formula $A = \frac{s^2\sqrt{3}}{4}$, where *s* represents the side length of the equilateral triangle.

$$A_{\text{triangle}} = \frac{s^2\sqrt{3}}{4}$$
$$A_{\text{triangle}} = \frac{(90)^2\sqrt{3}}{4}$$
$$A_{\text{triangle}} = \frac{8100\sqrt{3}}{4}$$
$$A_{\text{triangle}} = 2025\sqrt{3}$$

- Calculate the area of one segment.

$$A_{\text{segment}} = A_{\text{sector}} - A_{\text{triangle}}$$
$$A_{\text{segment}} = 1350\pi - 2025\sqrt{3}$$
$$A_{\text{segment}} \approx 733.75 \text{ square miles}$$



The overlap region is composed of two sectors and two segments. Determine the sum of the areas of these four figures.

$$\text{Total Area} = A_{\text{sector}} + A_{\text{sector}} + A_{\text{segment}} + A_{\text{segment}}$$
$$\text{Total Area} \approx 733.75 + 733.75 + 1350\pi + 1350\pi$$
$$\text{Total Area} \approx 9950 \text{ square miles}$$

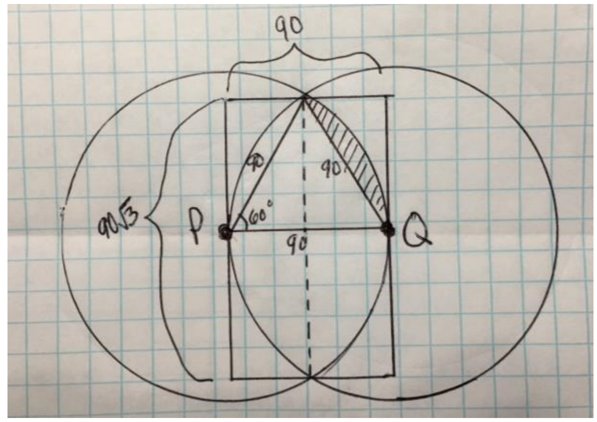
The area receiving the signal of both radio stations is about 9950 square miles.

Evaluating the reasonableness of the solution

Draw a rectangle around the overlap region that has the same height and width as the overlap region. The width of this rectangle is 90 miles and the height of this rectangle is $90\sqrt{3}$ miles (The height of the equilateral triangle can be calculated using the properties of special right triangles).

$90(90\sqrt{3}) = 8100\sqrt{3} \approx 14{,}029$ square miles

This rectangle is larger than the overlap region, so the area of the rectangle should be greater than the area of the overlap region. $14{,}029 > 9{,}950$, so the approximate area of 9,950 square miles for the overlap region is reasonable.

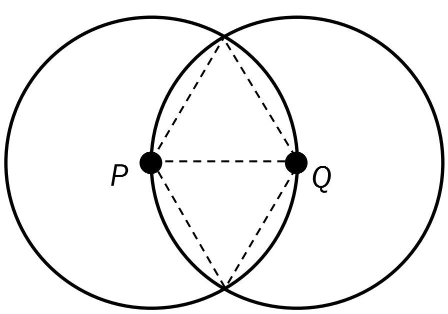


Look For...

- a correct application of the formula for the area of a sector
- recognition of the parts of a circle, including sectors, segments, and radii
- identification of an equilateral triangle created by three congruent radii
- reasoning about how to decompose the overlap region into non-overlapping parts

Differentiation: Simplified Task

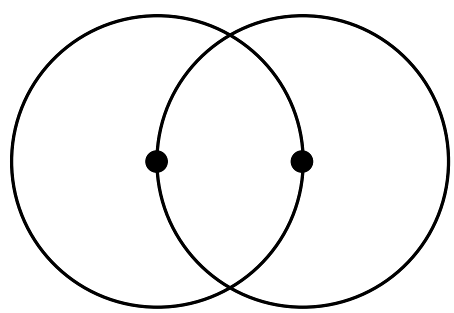
Two radio stations broadcast with the same intensity from two neighboring cities that are 90 miles apart. The signal from each station covers an area shaped like a circle. The signal is just strong enough to reach the transmitter from the other station so that the broadcast areas are congruent. What is the approximate area of the sector of circle *P* that contains point *Q*? Justify your reasoning.



Answer: *2700π square miles*

Differentiation: Enriching Task

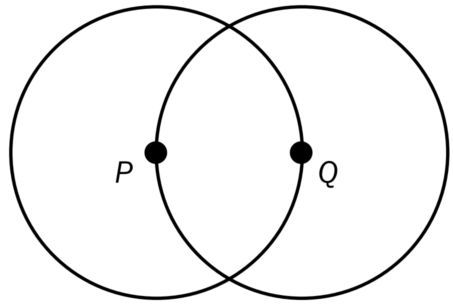
Two radio stations broadcast with the same intensity from two neighboring cities that are 90 miles apart. The signal from each station covers an area shaped like a circle. The signal is just strong enough to reach the transmitter from the other station so that the broadcast areas are congruent. What is the approximate area of the region that can listen to only one, but not both, of the radio stations? Justify your reasoning.



Answer: *approximately 40,494 square miles*

Cluster G.12: Circles

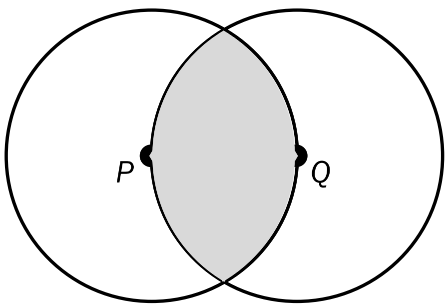
Scaffolded Task with Answers



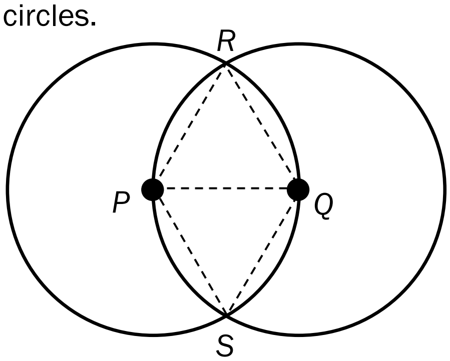
Two radio stations broadcast with the same intensity from two neighboring cities that are 90 miles apart. The signal from each station covers an area shaped like a circle. The signal is just strong enough to reach the transmitter from the other station so that the broadcast areas are congruent. What is the approximate area of the region that can listen to both radio stations?

Justify your reasoning.

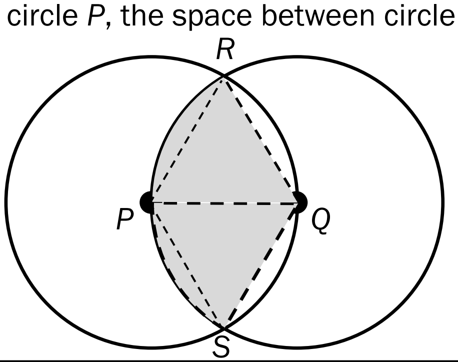
1. Identify the region in the diagram where both radio stations can be heard.



2. Label the top intersection point *R* and bottom intersection point *S*. Construct three radii from circle *P*. One should connect *P* and *Q* and two should connect *P* with the intersection points of the two circles. Also, construct two radii from point *Q* to the intersection points of the two circles.



3. Decompose the overlap region into two sectors of circle *Q* and two remaining segments of circle *P*, the space between circle *P* and the sector of circle *Q*.



4. Classify triangle *PQR* and triangle *PQS* using the lengths of the segments creating each triangle (Hint: All radii of a circle are congruent).
   Both triangles are equilateral triangles.

5. Determine the measure of angle *RQP* and angle *SQP*.
   Each angle measures 60°.

6. Determine the area of each sector of circle *Q*.
   $A_{\text{sector}} = \frac{n}{360}(\pi r^2)$
   $A_{sector} = \frac{60}{360}(\pi(90)^2)$
   $A_{sector} = \frac{1}{6}(8100)\pi$
   $A_{sector} = 1350\pi$ *square miles*

7. Determine the area of triangle *PQR*.
   $A_{\text{triangle}} = \frac{s^2\sqrt{3}}{4}$
   $A_{\text{triangle}} = \frac{(90)^2\sqrt{3}}{4}$
   $A_{\text{triangle}} = \frac{8100\sqrt{3}}{4}$
   $A_{\text{triangle}} = 2025\sqrt{3}$

8. Determine the area of the segment of the circle bounded by $\overline{RQ}$.
   $A_{\text{segment}} = A_{\text{sector}} - A_{\text{triangle}}$
   $A_{segment} = 1350\pi - 2025\sqrt{3}$
   $A_{segment} \approx 733.75$ *square miles*

9. Determine the area of the segment of the circle bounded by $\overline{RQ}$.
   $A_{segment} = A_{sector} - A_{triangle}$
   $A_{segment} = 1350\pi - 2025\sqrt{3}$
   $A_{segment} \approx 733.75$ *square miles*

10. Determine the total area of the overlap region.
   Total Area $= A_{sector} + A_{sector} + A_{segment} + A_{segment}$
   Total Area $\approx 733.75 + 733.75 + 1350\pi + 1350\pi$
   Total Area ≈ 9950 *square miles*

Name ______________________________ Date ____________________

Performance Task: G.12C

Area of Sectors of Circles: On the Radio

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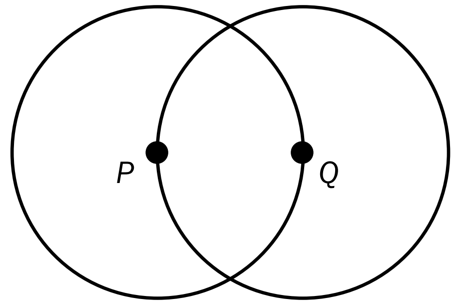

Justify your reasoning.

Procedural	0	1	2
Conceptual	0	1	2
Communication	0	1	2

Total points: _______

Name ______________________________ Date ______________________

Performance Task: G.12C
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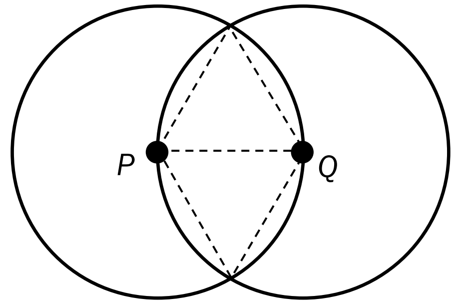

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Performance Task: G.12C

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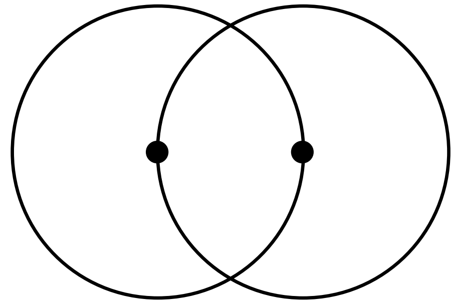

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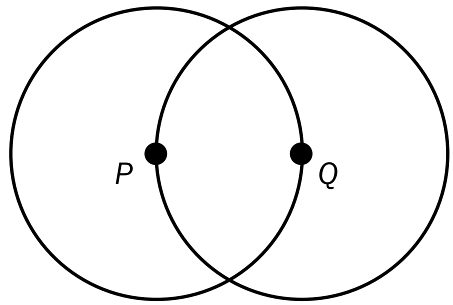

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Name ______________________________ Date ____________________

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   $A_{\text{segment}} = A_{\text{sector}} - A_{\text{triangle}}$

9. Determine the area of the segment of the circle bounded by $\overline{RQ}$.

10. Determine the total area of the overlap region.