Cluster 2A.7: Number and Algebraic Methods

2A.7D: Linear Factors of Polynomial Functions: Sorting Diagram

Focusing TEKS

2A.7D Number and algebraic methods. The student applies mathematical processes to simplify and perform operations on expressions and to solve equations. The student is expected to determine the linear factors of a polynomial function of degree three and of degree four using algebraic methods.

Additional TEKS:

2A.7C The student is expected to determine the quotient of a polynomial of degree three and of degree four when divided by a polynomial of degree one and of degree two.

A.10E The student is expected to factor, if possible, trinomials with real factors in the form $ax^2 + bx + c$, including perfect square trinomials of degree two.

Focusing Mathematical Process

2A.1D The student is expected to communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

2A.1F Analyze mathematical relationships to connect and communicate mathematical ideas.

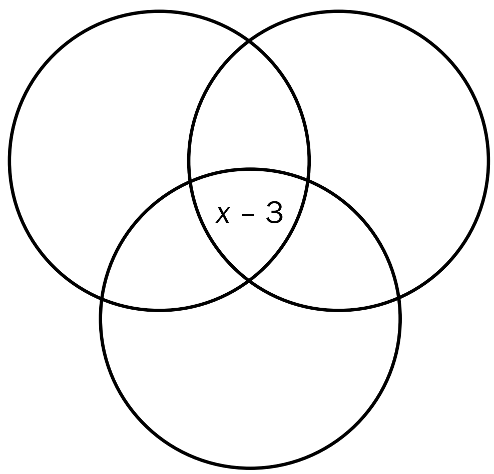
2A.1G Display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Performance Task

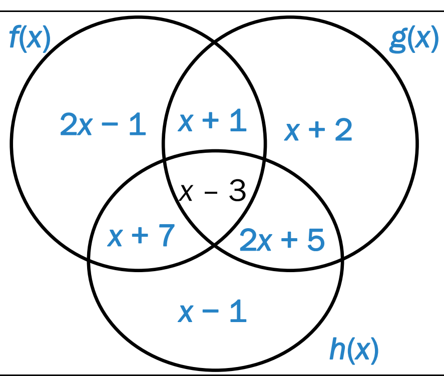
Determine the linear factors of each of the following polynomial functions.

- $f(x) = 2x^4 + 9x^3 - 39x^2 - 25x + 21$
- $g(x) = 2x^4 + 5x^3 - 14x^2 - 47x - 30$
- $h(x) = 2x^4 + 11x^3 - 35x^2 - 83x + 105$

Use the linear factors to complete the Venn diagram. Each circle represents one of the polynomial functions and the overlap region among two or more circles represents a linear factor that is shared by those polynomial functions.



Answer:



Cluster 2A.7: Number and Algebraic Methods

Mathematically Speaking…

In this task, students will need to completely factor each of three fourth-degree polynomial functions into a product of four linear factors. Students are given one linear factor, $x - 3$, that is common to all three functions. From there, students may use a variety of methods to determine the remaining linear factors.

- Use a strategy such as synthetic division or long polynomial division in order to determine the cubic polynomial function and then resulting quadratic function.
- Use a graph to determine the x-intercepts (roots) of each function and then use the Zero Product Property to write the linear factors.
- Apply the rational root theorem to determine additional linear factors.

Students will complete the Venn diagram once the linear factors have been identified by determining which linear factors are common to which polynomial functions. Some linear factors are shared with two polynomial functions and some are unique to a particular polynomial function.

Possible Solution

First, determine the linear factors of each of the given functions.

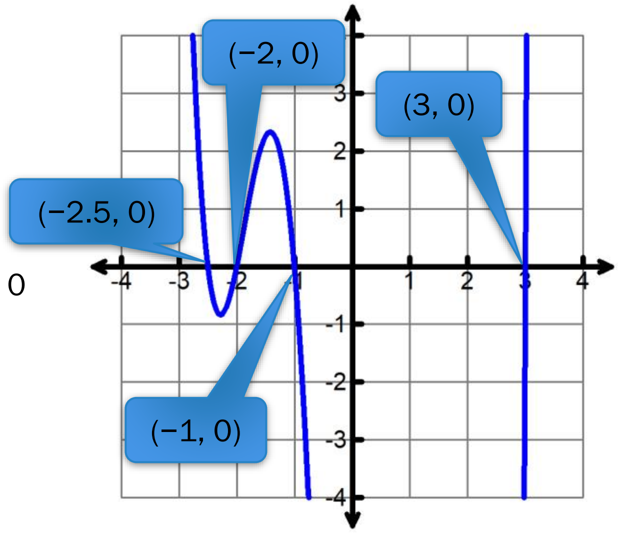
Begin with $f(x) = 2x^4 + 9x^3 - 39x^2 - 25x + 21$. We know that one factor is $x - 3$, so use synthetic division to determine the third-degree function that remains after $x - 3$ is divided out of the fourth-degree function.

$$\begin{array}{r|rrrrr} 3 & 2 & 9 & -39 & -25 & 21 \\ & & 6 & 45 & 18 & -21 \\ \hline & 2 & 15 & 6 & -7 & 0 \end{array} \quad \Rightarrow \quad f(x) = (x - 3)(2x^3 + 15x^2 + 6x - 7)$$

$$\begin{array}{r|rrrr} -7 & 2 & 15 & 6 & -7 \\ & & -14 & -7 & 7 \\ \hline & 2 & 1 & -1 & 0 \end{array} \quad \Rightarrow \quad f(x) = (x - 3)(x + 7)(2x^2 + x - 1)$$

$$\begin{array}{r|rrr} -1 & 2 & 1 & -1 \\ & & -2 & 1 \\ \hline & 2 & -1 & 0 \end{array} \quad \Rightarrow \quad f(x) = (x - 3)(x + 7)(x + 1)(2x - 1)$$

Next, determine the linear factors of $g(x) = 2x^4 + 5x^3 - 14x^2 - 47x - 30$. If you graph $g(x)$, you can determine the x-intercepts and thus the roots of $g(x)$. With the roots, work backwards using the Zero Product Property to determine the linear factors of $g(x)$.



$x = 2.5$	$x = -2$	$x = -1$	$x = 3$
$x + 2.5 = 0$	$x + 2 = 0$	$x + 1 = 0$	$x - 3 = 0$
$x + \frac{5}{2} = 0$			
$2x + 5 = 0$			

So, $g(x) = (2x + 5)(x + 2)(x + 1)(x - 3)$.

Finally, determine the linear factors of $h(x) = 2x^4 + 11x^3 - 35x^2 - 83x + 105$. Again, you know that one factor is $x - 3$, so long division to determine the remaining third-degree function.

$$
\begin{array}{r}
2x^3 + 17x^2 + 16x - 35 \\
x - 3 \overline{)\, 2x^4 + 11x^3 - 35x^2 + 83x + 105} \\
-(2x^4 - 6x^3) \\
\hline
17x^3 - 35x^2 \\
-(17x^3 - 51x^2) \\
\hline
16x^2 + 83x \\
-(16x^2 - 48x) \\
\hline
-35x + 105 \\
-(-35x - 105) \\
\hline
0
\end{array}
$$

$h(x) = (x - 3)(2x^3 + 17x^2 + 16x - 35)$

The rational root theorem tells you that any rational roots of a polynomial function are a rational number where the numerator is a factor of the constant term and the denominator is a factor of the coefficient of the highest degree. For the cubic factor of $h(x)$:

$$\text{Rational Root Theorem: } \frac{\text{Factors of 35}}{\text{Factors of 2}} = \frac{\pm 1, 5, 7, 35}{\pm 1, 2}$$

There are many possibilities for the next root. Here is where trial and error becomes a part of the solution strategy. Try $\frac{7}{1} = 7$ with synthetic division.

$$
\begin{array}{r|rrrr}
7 & 2 & 17 & 16 & -35 \\
 & & 14 & 217 & 1631 \\
\hline
 & 2 & 31 & 233 & 1596
\end{array}
$$

The final number in the synthetic quotient is not 0, so 7 is not a root of the cubic factor and $x - 7$ is not a linear factor of $h(x)$. Try -7.

−7	2	17	16	−35
		−14	−21	35
	2	3	−5	0

→ $h(x) = (x - 3)(x + 7)(2x^2 + 3x - 5)$

The final number in the synthetic quotient is 0, so −7 is a root of the cubic factor and $x + 7$ is a linear factor of $h(x)$. Continue factoring the quadratic root of $h(x)$. Using factors of $2x^2$ and −5, the quadratic root factors to $(2x + 5)(x - 1)$.

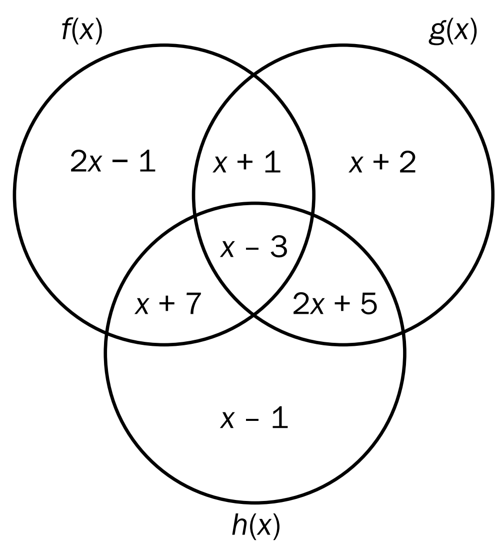
$h(x) = (x - 3)(x + 7)(2x + 5)(x - 1)$

Now, make a list of which linear factors are associated with which function.

$f(x)$	$g(x)$	$h(x)$
$(x - 3)$	$(x - 3)$	$(x - 3)$
$(x + 1)$	$(x + 1)$	$(x + 7)$
$(x + 7)$	$(2x + 5)$	$(2x + 5)$
$(2x - 1)$	$(x + 2)$	$(x - 1)$

Construct a Venn diagram of three overlapping circles.

- Label each circle $f(x)$, $g(x)$, and $h(x)$.
- $x - 3$ is in the center of the diagram so is a linear factor of all three functions.
- $x + 1$ is a linear factor of both $f(x)$ and $g(x)$, so place it in the overlap region of those two circles.
- $x + 7$ is a linear factor of both $f(x)$ and $h(x)$, so place it in the overlap region of those two circles.
- $2x + 5$ is a linear factor of both $g(x)$ and $h(x)$, so place it in the overlap region of those two circles.
- The remaining linear factors are unique to their respective functions.
  - Place $2x - 1$ in the non-overlapping region of $f(x)$.
  - Place $x + 2$ in the non-overlapping region of $g(x)$.
  - Place $x - 1$ in the non-overlapping region of $h(x)$.



Look For…

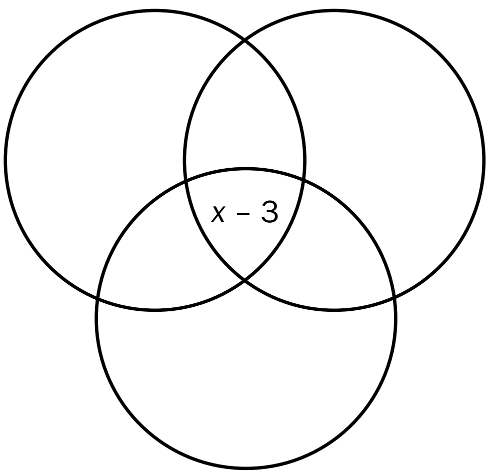
- justification of the method of factoring
- correct application of the method of factoring
- connections among roots of the function and linear factors
- reduction of the fourth-degree polynomials by one degree with the extraction of each linear factor
- correct reasoning for the placement of each linear factor in the Venn diagram

Differentiation: Simplified Task

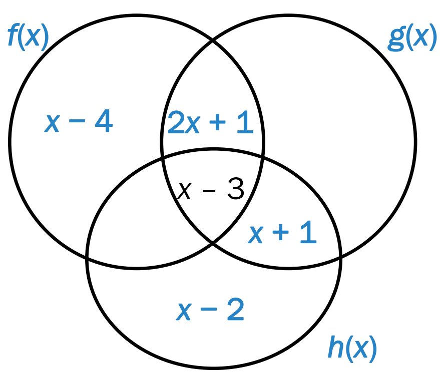
Determine the linear factors of each of the following polynomial functions.

- $f(x) = 2x^3 - 13x^2 + 17x + 12$
- $g(x) = 2x^3 - 3x^2 - 8x - 3$
- $h(x) = x^3 + 2x^2 - 5x - 6$

Use the linear factors to complete the Venn diagram. Each circle represents one of the polynomial functions and the overlap region among two or more circles represents a linear factor that is shared by those polynomial functions.



Answer:

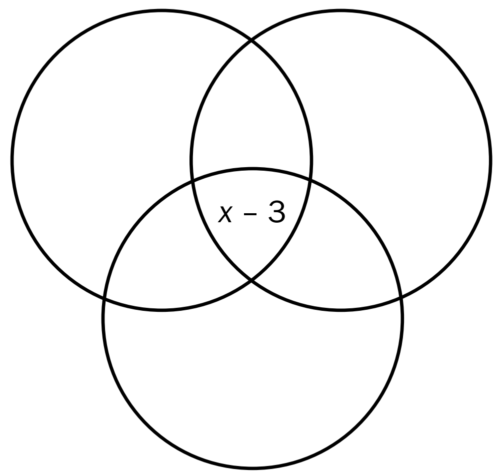


Differentiation: Enriching Task

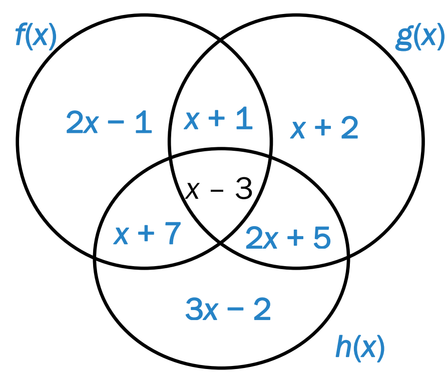
Determine the linear factors of each of the following polynomial functions.

- $f(x) = 2x^4 + 9x^3 - 39x^2 - 25x + 21$
- $g(x) = 2x^4 + 5x^3 - 14x^2 - 47x - 30$
- $h(x) = 6x^4 + 35x^3 - 92x^2 - 271x + 210$

Use the linear factors to complete the Venn diagram. Each circle represents one of the polynomial functions and the overlap region among two or more circles represents a linear factor that is shared by those polynomial functions.



Answer:



Cluster 2A.7: Number and Algebraic Methods

Scaffolded Task with Answers

Determine the linear factors of each of the following polynomial functions.

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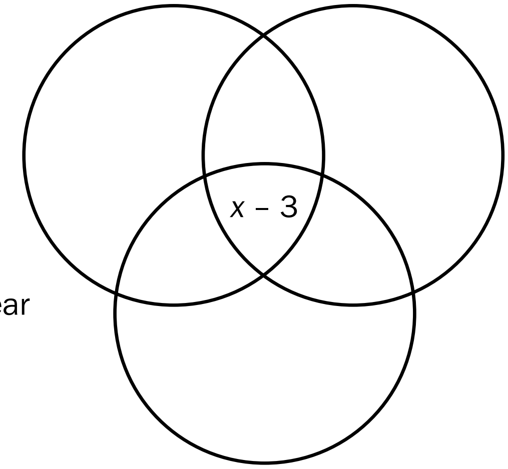


Use the linear factors to complete the Venn diagram. Each circle represents one of the polynomial functions and the overlap region among two or more circles represents a linear factor that is shared by those polynomial functions.

1. Determine the linear factors of $f(x)$.
   $f(x) = (x - 3)(x + 7)(x + 1)(2x - 1)$

2. Determine the linear factors of $g(x)$.
   $g(x) = (2x + 5)(x + 2)(x + 1)(x - 3)$

3. Determine the linear factors of $h(x)$.
   $h(x) = (x - 3)(x + 7)(2x + 5)(x - 1)$

4. List the factors of each function.

$f(x)$	$g(x)$	$h(x)$
$(x - 3)$	$(x - 3)$	$(x - 3)$
$(x + 1)$	$(x + 1)$	$(x + 7)$
$(x + 7)$	$(2x + 5)$	$(2x + 5)$
$(2x - 1)$	$(x + 2)$	$(x - 1)$

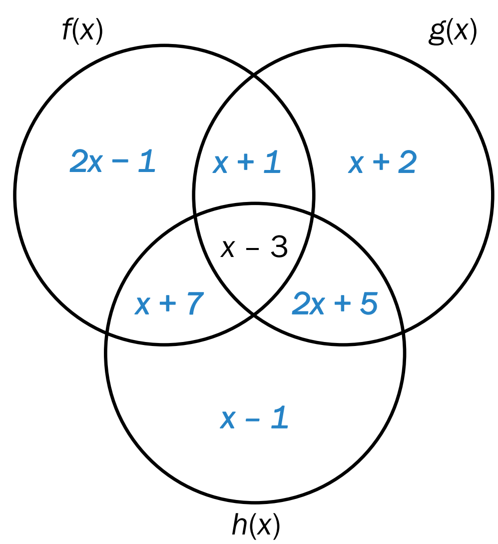
The factor $x - 3$ is in the center of the diagram so is a linear factor of all three functions.

5. Identify the linear factor shared by both $f(x)$ and $g(x)$. Place it in the overlap region of those two circles.

6. Identify the linear factor of both $f(x)$ and $h(x)$ and place it in the overlap region of those two circles.

7. Identify the linear factor of both $g(x)$ and $h(x)$ and place it in the overlap region of those two circles.

8. Place the remaining linear factors that are unique to their respective functions in the non-overlapping regions of $f(x)$, $g(x)$, and $h(x)$.



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Performance Task: 2A.7D

Linear Factors of Polynomial Functions: Sorting Diagram

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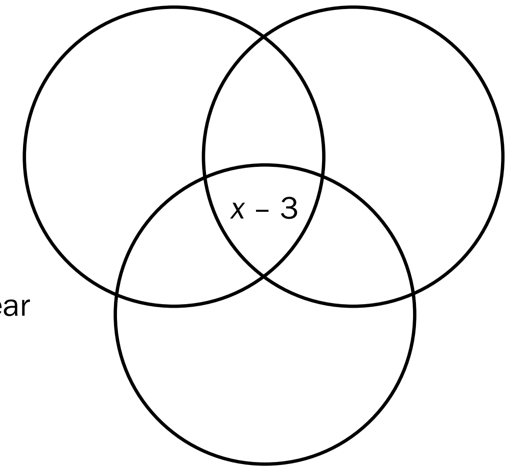

Use the linear factors to complete the Venn diagram. Each circle represents one of the polynomial functions and the overlap region among two or more circles represents a linear factor that is shared by those polynomial functions.

Procedural	0	1	2
Conceptual	0	1	2
Communication	0	1	2

Total points: _______

Name ____ Date ___________________________

Performance Task: 2A.7D

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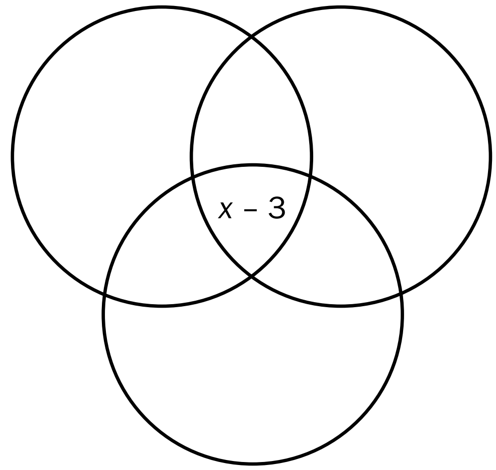

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Performance Task: 2A.7D
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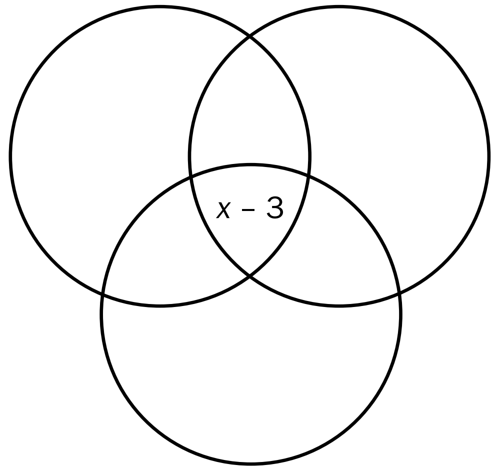

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Name ____ Date ______________________

Performance Task: 2A.7D

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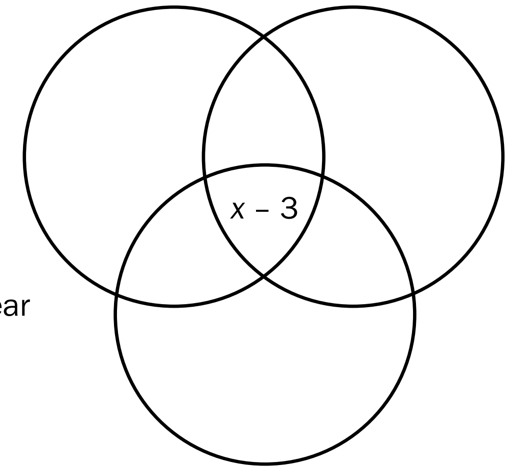

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1. Determine the linear factors of $f(x)$.

2. Determine the linear factors of $g(x)$.

Name ____ Date ______________________

3. Determine the linear factors of $h(x)$.

4. List the factors of each function.

$f(x)$	$g(x)$	$h(x)$
$(x - 3)$	$(x - 3)$	$(x - 3)$

The factor $x - 3$ is in the center of the diagram so is a linear factor of all three functions.

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