Cluster 2A.4: Quadratic and Square Root Functions, Equations, and Inequalities

2A.4F: Square Root Equations – Lighthouse Tour

Focusing TEKS

2A.4F Quadratic and square root functions, equations, and inequalities. The student applies mathematical processes to understand that quadratic and square root functions, equations, and quadratic inequalities can be used to model situations, solve problems, and make predictions. The student is expected to solve quadratic and square root equations.

Additional TEKS:

2A.4E The student is expected to formulate quadratic and square root equations using technology given a table of data.

Focusing Mathematical Process

2A.1A Apply mathematics to problems arising in everyday life, society, and the workplace.

2A.1B Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution.

2A.1D The student is expected to communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

Performance Task

Jaqueece likes to visit lighthouses. He keeps a journal of lighthouses that he has visited, the height of the lighthouse, and the distance of the farthest object he could see from the top of the lighthouse on a clear day. A table from his journal is shown.

Lighthouse	Port Isabel, Texas	Aransas Pass Light Station, Texas	Portland Head Light, Maine	Crooked River, Florida	Bolivar Point, Texas
Height in feet	57	68	80	100	116
Distance in miles	13.2	14.4	15.7	17.5	18.8

Jaqueece is going to visit Cape Hatteras Lighthouse in North Carolina and knows from a blog post that on a clear day, you can see about 24.3 miles from the top of the lighthouse. He also knows that the distance you can see from the top of a lighthouse depends on the height of the lighthouse and can be modeled with a square root function. About how tall is Cape Hatteras Lighthouse? Justify your reasoning.

Answer: *Cape Hatteras Lighthouse is about 193 feet tall.*

Cluster 2A.4: Quadratic and Square Root Functions, Equations, and Inequalities

Mathematically Speaking...

In this task, students need to use a table of data to write a square root function. There are several ways students can do this.

- Begin with the parent function, $y = \sqrt{x}$, and use transformations to fit a curve to a graph of the data.
- Use regression through technology.

Once a function has been determined, students will need to write a related equation using the given value of the dependent variable in the problem. Once the equation has been written, students may use a variety of strategies to solve it, including inverse operations, graph the function and $y = 23.4$ and determine the x-coordinate of the intersection point, or use a table of function values to determine the input value generating an output value of 23.4.

Students may also use rates of change in the table to estimate a solution to the problem.

Possible Solution

Analyzing the given information

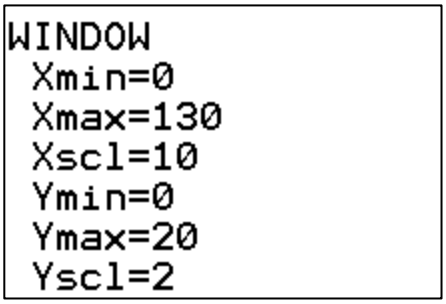
You are given a table of values and told that the distance seen from the top of a lighthouse depends on the height of the lighthouse and that the relationship is a square root function. The independent variable, x, is the height of the lighthouse in feet and the dependent variable, y, is the distance you can see in miles.

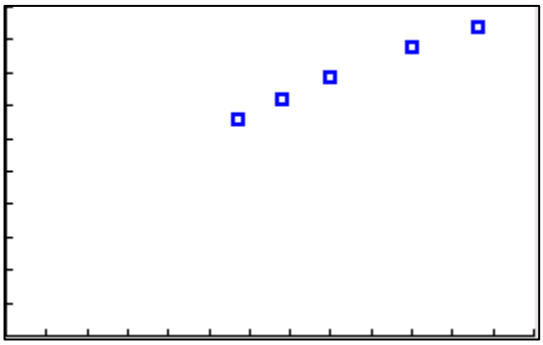
Formulating a plan

- Use the table to write a square root function, y, in terms of x.
- Write a related equation for a lighthouse where you can see 24.3 miles from its top.
- Solve the equation for x.

Determining a solution

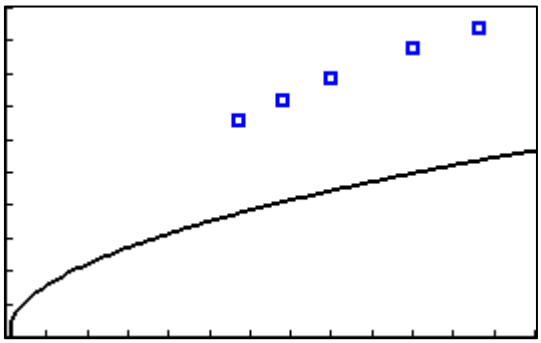
Make a scatterplot of the data, distance, y, versus height, x.



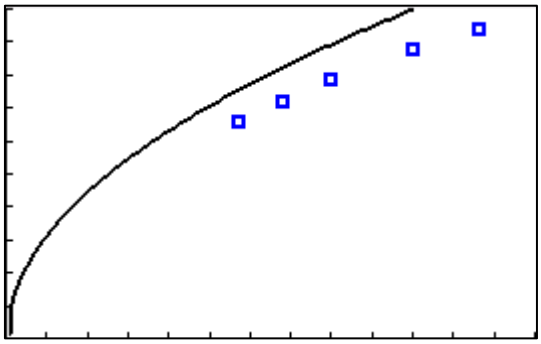


Cluster 2A.4: Quadratic and Square Root Functions, Equations, and Inequalities

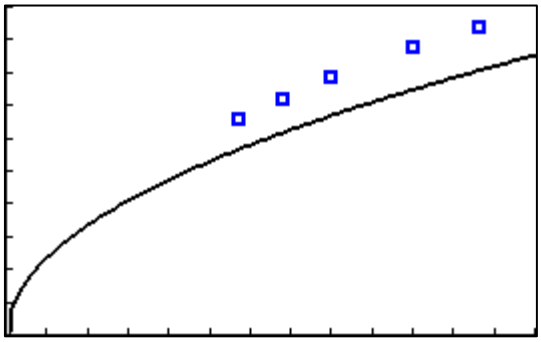
Graph the square root parent function, $y = \sqrt{x}$, on top of the scatterplot.



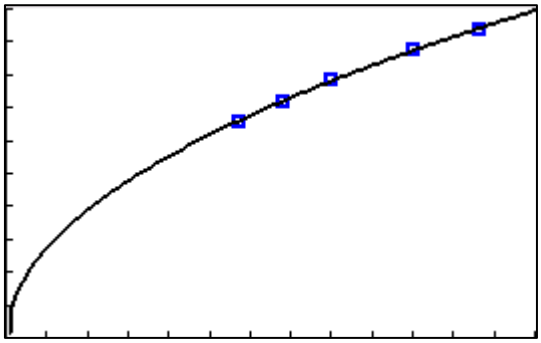
The scatterplot representing the data set is higher along the *y*-axis, so vertically stretch the graph of the parent function by multiplying $\sqrt{x}$ by a factor greater than 1. Graph $y = 2\sqrt{x}$.



The graph of $y = 2\sqrt{x}$ is now above the scatterplot, so try a multiplier that is less than 2 but still greater than 1, say, $y = 1.5\sqrt{x}$.



The graph of $y = 1.5\sqrt{x}$ is now beneath the scatterplot so try $y = 1.75\sqrt{x}$.



The function $y = 1.75\sqrt{x}$ appears to connect the points in the scatterplot, so this function model will work to describe the relationship between *x*, the height of the lighthouse in feet, and *y*, the distance of the farthest object you can see in miles. Test the fit by comparing a table of function values to the actual data from the original table.

Lighthouse	Height in feet, x	Distance in miles	$y = 1.75\sqrt{x}$
Port Isabel, Texas	57	13.2	13.212
Aransas Pass Light Station, Texas	68	14.4	14.431
Portland Head Light, Maine	80	15.7	15.652
Crooked River, Florida	100	17.5	17.500
Bolivar Point, Texas	116	18.8	18.848

Jaqueece knows that you can see about 24.3 miles from the top of Cape Hatteras Lighthouse. This number is the distance you can see from the lighthouse, which is the dependent variable, y. Substitute $y = 24.3$ into the function, $y = 1.75\sqrt{x}$.

$$24.3 = 1.75\sqrt{x}$$

Solve this related equation for x.

$$24.3 = 1.75\sqrt{x}$$

$$\frac{24.3}{1.75} = \frac{1.75\sqrt{x}}{1.75}$$

$$13.8857 \approx \sqrt{x}$$

Use the symbol ≈, "approximately equal to," instead of an equal sign because the quotient is rounded.

$$(13.8857)^2 \approx \left(\sqrt{x}\right)^2$$

$$193 \approx x$$

Cape Hatteras Lighthouse is approximately 193 feet tall.

Look For...

- method of determining the function from a table (transformation, regression, etc.)
- justification of why the function fits the data
- interpretation of the value of 24.3 as the dependent value of the function
- correct application of inverse operations to solve a square root equation

Cluster 2A.4: Quadratic and Square Root Functions, Equations, and Inequalities

Differentiation: Simplified Task

Jaqueece likes to visit lighthouses. He keeps a journal of lighthouses that he has visited, the height of the lighthouse, and the distance of the farthest object he could see from the top of the lighthouse on a clear day. A table from his journal is shown.

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Crooked River, Florida	100	17.5
Bolivar Point, Texas	116	18.8

Jaqueece is going to visit Piedras Blancas Light in California and knows from a blog post that on a clear day, you can see 15 miles from the top of the lighthouse. He also knows that the distance you can see from the top of a lighthouse depends on the height of the lighthouse and can be modeled with a square root function. About how tall is Piedras Blancas Light? Justify your reasoning.

Answer:
Piedras Blancas Light is about 74 feet tall.

Differentiation: Enriching Task

Jaqueece likes to visit lighthouses. He keeps a journal of lighthouses that he has visited, the height of the lighthouse, and the distance of the farthest object he could see from the top of the lighthouse on a clear day. A table from his journal is shown.

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Bolivar Point, Texas	116	18.8

This summer, Jaqueece will visit Cape Hatteras Lighthouse in North Carolina and Harbour Town Light in South Carolina. According to a blog post, you can see about 24.3 miles from the top of Cape Hatteras Lighthouse and about 16.6 miles from the top of Harbour Town Light. The distance you can see from the top of a lighthouse depends on the height of the lighthouse and can be modeled with a square root function. About how much taller is Cape Hatteras Lighthouse than Harbour Town Light? Justify your reasoning.

Answer: Cape Hatteras Lighthouse is about 100 feet taller than Harbour Town Light.

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Scaffolded Task with Answers

Jaqueece likes to visit lighthouses. He keeps a journal of lighthouses that he has visited, the height of the lighthouse, and the distance of the farthest object he could see from the top of the lighthouse on a clear day. A table from his journal is shown.

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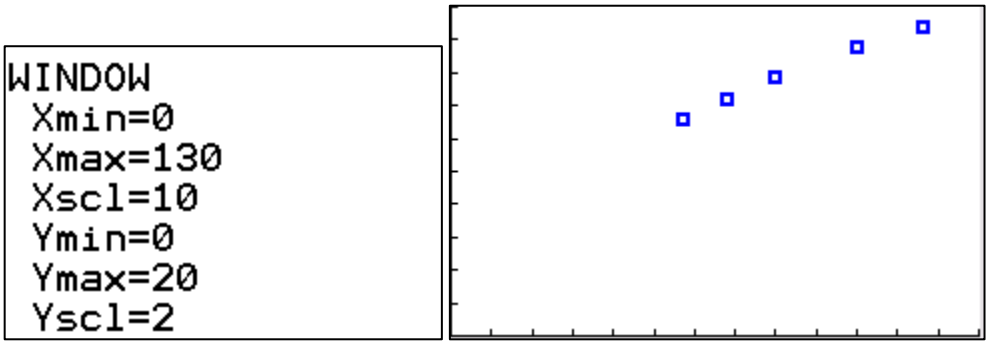
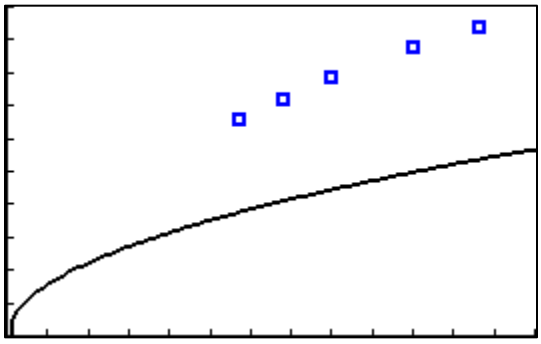
1. Identify the independent and dependent variables in this situation.
   The height of the lighthouse is the independent variable.
   The distance seen from the lighthouse is the dependent variable.

2. Use graphing technology to make a scatterplot of distance versus height.

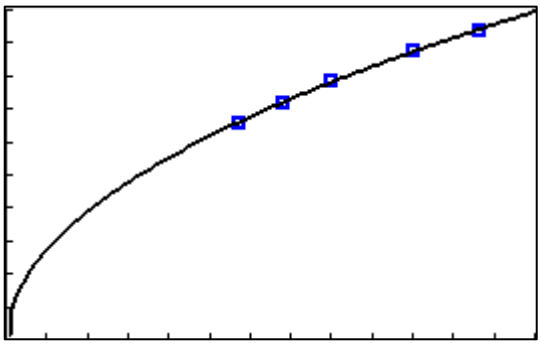


3. Graph the parent square root function, $y = \sqrt{x}$, on top of the scatterplot.



4. Use transformations to make the graph of the square root function line up on top of the scatterplot. Record the equation of the curve that fits the scatterplot.

   $y = 1.75\sqrt{x}$



5. Compare a table of values for the function to the data in the original table. How close are the predicted distance values from the function to the actual distances seen from each lighthouse?

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   If you round the predicted values to the nearest tenth, they are the same as the observed distances.

6. An object 24.3 miles away can be seen from Cape Hatteras Lighthouse. Is this number a part of the independent variable or dependent variable?

   dependent variable

7. Use 24.3 miles to write an equation related to the function modeling the data in the table.

   $24.3 = 1.75\sqrt{x}$

8. Solve the equation.

   $24.3 = 1.75\sqrt{x}$

   $\frac{24.3}{1.75} = \frac{1.75\sqrt{x}}{1.75}$

   $13.8857 \approx \sqrt{x}$

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Name _____ Date _____

Performance Task: 2A.4F
Square Root Equations: Lighthouse Tour

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Procedural	0	1	2
Conceptual	0	1	2
Communication	0	1	2

Total points: _____

Name _____ Date _____

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