MULTIPLYING POLYNOMIALS

A.10B **The student is expected to multiply polynomials of degree one and degree two.**

TELL ME MORE...

A **polynomial** is an expression that is a sum of several terms. Polynomials may contain multiple variables, real number coefficients, and whole number exponents. Polynomials may not contain division with variables or non-whole number exponents.

Types of polynomials are named by the number of terms contained in the polynomial. **Monomials** have one term, **binomials** have two terms, and **trinomials** have three terms. Polynomials, of course, may have four or more terms as well.

The **degree** of a polynomial is its greatest exponent. For example if the term in the polynomial with the greatest exponent is $7x^8$ then the polynomial is of *degree eight*. The polynomial $5.4x^2 - 8.3x + 11$ is of *degree two* since the term with the greatest exponent has an exponent of two.

To multiply polynomials symbolically, use the distributive property. Multiply each term in the first factor by each term in the second factor. Then, use the properties of algebra to simplify the expression representing the product.

> **Monomials** have one term.
>
> Examples: 4, $4x$, $\frac{2}{3}x^2y^3$
>
> **Binomials** have two terms.
>
> Examples: $x + 4$, $3b - 4c$, $5xy + 1.8x^2y$
>
> **Trinomials** have three terms.
>
> Examples: $x^2 - \frac{2}{3}x + 4$, $3b^4 - 4c + 89d^7$

EXAMPLES

EXAMPLE 1: Multiply $(5x - 7)(x + 2)$.

STEP 1 Use the distributive property to multiply the first term in the first factor, 5x, by both terms in the second factor.

$$(5x - 7)(x + 2)$$
$$5x(x) + 5x(2)$$
$$5x^2 + 10x$$

$\mathbf{5x^2 + 10x}$

STEP 2 Use the distributive property to multiply the second term in the first factor, 5x, by both terms in the second factor.

$$(5x - 7)(x + 2)$$
$$-7(x) - 7(2)$$
$$-7x - 14$$

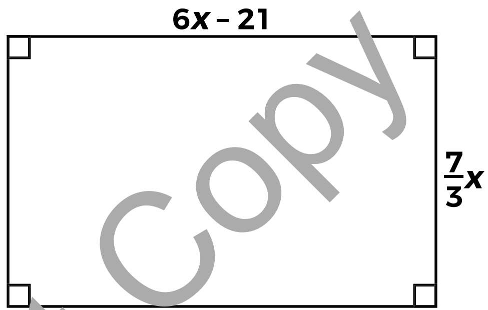
$\mathbf{-7x - 14}$

STEP 3 Combine the two partial products.

$$(5x - 7)(x + 2) = [5x^2 + 10x] + [-7x - 14] = 5x^2 + 10x - 7x - 14$$
$$(5x - 7)(x + 2) = 5x^2 + 3x - 14$$

$5x^2 + 3x - 14$

EXAMPLE 2: Ian marked the boundary of a beach volleyball court as shown. The dimensions of the court are given in meters. Write an expression that represents the area of the beach volleyball court in square meters.



STEP 1 The area of a rectangle can be calculated using the formula $A = lw$. Let l and w each represent one dimension. Substitute the dimensions into the area formula.

$$A = lw$$
$$A = \left(\frac{7}{3}x\right)(6x - 21)$$

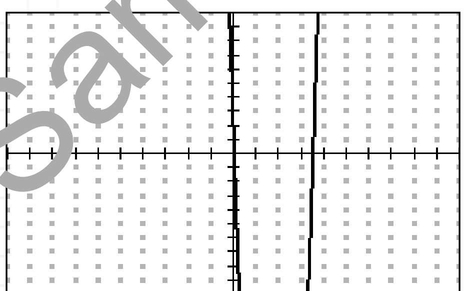
$\left(\frac{7}{3}x\right)(6x - 21)$

STEP 2 Use the distributive property to multiply the first factor by both terms of the second factor.

$$\left(\frac{7}{3}x\right)(6x - 21)$$
$$\left(\frac{7}{3}x\right)(6x) + \left(\frac{7}{3}x\right)(-21)$$
$$14x^2 - 49x$$

$14x^2 - 49x$

STEP 3 Use graphing technology to check your answer. Graph $Y1 = \left(\frac{7}{3}x\right)(6x - 21)$, the product of the two dimensions, and $Y2 = 14x^2 - 49x$, the simplified product. If the graphs coincide, then the expressions are equivalent. You can also use tables to verify equivalence.



X	Y1	Y2		
0	0	0		
1	-35	-35		
2	-42	-42		
3	-21	-21		
4	28	28		
5	105	105		
6	210	210		
7	343	343		
8	504	504		
9	693	693		
10	910	910		

The graphs coincide and table shows equivalent y-values for each x-value. The expressions are equivalent

YOU TRY IT!

The area of a triangle is determined using the formula $A = \frac{1}{2}bh$, where b represents the base length and h represents the height of the triangle. Write an expression for the area of a triangle with a base length of $(5x - 4)$ feet and a height of $(3x + 7)$ feet.

$A = \frac{1}{2}$ (____________)(____________)

Use the distributive property:

Simplify the expression:

EXAMPLE 3: For what value of k will the graphs of $y = 3x^2 - 24x + k$ and $y = 3(x - 4)^2 + 15$ be the same? Record your answer and fill in the bubbles on your answer document.

STEP 1 Rewrite $y = 3(x - 4)^2 + 15$ in standard form. Use the distributive property and other properties of algebra to simplify the expression.

$$y = 3(x - 4)^2 + 15$$
$$y = 3(x - 4)(x - 4) + 15$$
$$y = 3(x^2 - 8x + 16) + 15$$
$$y = 3x^2 - 24x + 48 + 15$$
$$y = 3x^2 - 24x + 63$$

$\boldsymbol{y = 3x^2 - 24x + 63}$

STEP 2 Compare the two functions to determine the value of k.

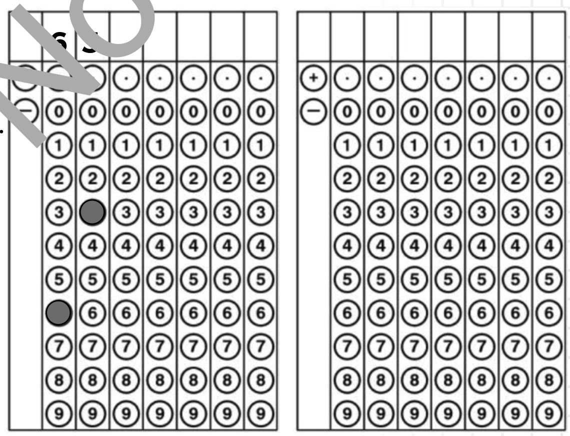
$$y = 3x^2 - 24x + k$$
$$y = 3x^2 - 24x + 63$$

Since the x^2 term and x term are the same in both functions, the constant term should also be the same in both functions for the functions to be equivalent and their graphs the same.

$\boldsymbol{k = 63}$

STEP 3 Since the question is a gridded response question, enter your response on the grid provided. Practice using the grid with the instructions.

1. Record a 6 in the first column containing numbers. Record a 3 in the next column.
2. Bubble the 6 beneath the numeral 6. Bubble the 3 beneath the numeral 3.



PRACTICE

Multiply the following pairs of polynomials.

1. $3x$ and $(4x - 5)$

2. $\frac{1}{2}x$ and $8x$

3. $1.5x$ and $(4x^2 - 10x + 20)$

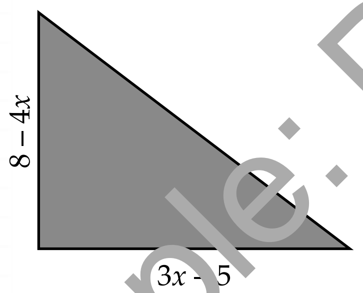
4. $(3m - 7)$ and $(11m + 2)$

5. $(7b + 8)$ and $(2b^2 - 5b + 8)$

6. $(3z^2 - 6z + 10)$ and $(z^2 + 2z - 1)$

7. On a recent road trip, Patrick traveled at an average rate of $3x + 5$ miles per hour and reached his destination in $4x - 1$ hours. Write an expression in standard form that represents the distance Patrick traveled ($d = rt$).

8. The diagram below shows a triangular garden with dimensions as marked.



The area of a triangle is found using the formula $A = \frac{1}{2}bh$ where A is the area, b is the base length of the triangle, and h is the height of the triangle. Write a trinomial expression to represent the area of the garden in square units.

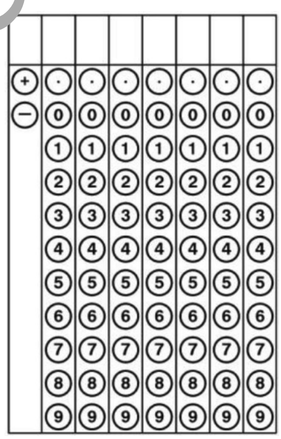
9. The bottom of Cinnamon's dog crate has a length 6 inches greater than its width, w. Cinnamon's owner Anna plans to get Cinnamon a new crate that is 4 inches larger on both the length and width. Write an expression in standard form that represents the area, A, of Cinnamon's new crate in terms of w, the width of her current crate?

10. The area of a trapezoid is found using the formula $A = \frac{h}{2}(b_1 + b_2)$, where A is the area of the trapezoid, h is the height, and b_1 and b_2 are the lengths of the trapezoid's bases. What expression in standard form represents the area of a trapezoid with bases $(x - 6)$ and $(3x + 4)$ inches long and a height of $5x$ inches?

11. What degree 4 expression is equivalent to $3a(4a^2 - a + 12)(7 - a)$?

12. Write an expression for the area of a rectangle with a length of $(x + 5)^2$ feet and a width of $(3 - x)$ feet.

13. The surface area of a cube can be calculated using the formula $A = 6s^2$, where s represents the edge length of the cube. If the edge length of a cube is $(2x + 7)$ centimeters, the surface area of the cube is equal to $24x^2 + 168x + c$. What is the value of c? Record your answer and fill in the bubbles on your answer document.



14. A packing box has a length of $(2x + 3)$ units, a width of $(4 - x)$ units, and a height of $(3x - 4)$ units. The volume of the box can be determined by using the formula $V = Bh$ where V is the volume, B is the area of the base of the box, and h is the height of the box. What expression can be used to find V, the volume of the box in terms of x in simplified form?

A $V = -6x - 48$

B $V = -6x^3 + 8x^2 + 36x - 48$

C $V = -6x^3 + 41x^2 - 8x - 48$

D $V = -6x^3 + 23x^2 + 16x - 48$

15. Kami was simplifying the expression $(2x - 3)(4x + 5)$ on a math exam. She got the answer $8x - 15$ but that answer was not one of the choices. What mistake did Kami make in simplifying and what should her answer have been?

F She should have added like terms to get $6x + 2$.

G She didn't fully distribute to get $8x^2 - 2x - 15$.

H She didn't combine middle terms correctly to get $8x^2 + 22x - 15$.

J She made a sign error and should have gotten $8x + 15$.