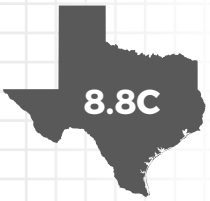
# SOLVING ONE-VARIABLE EQUATIONS




**The student is expected to model and solve one-variable equations with variables on both sides of the equal sign that represent mathematical and real-world problems using rational number coefficients and constants.**

## TELL ME MORE...

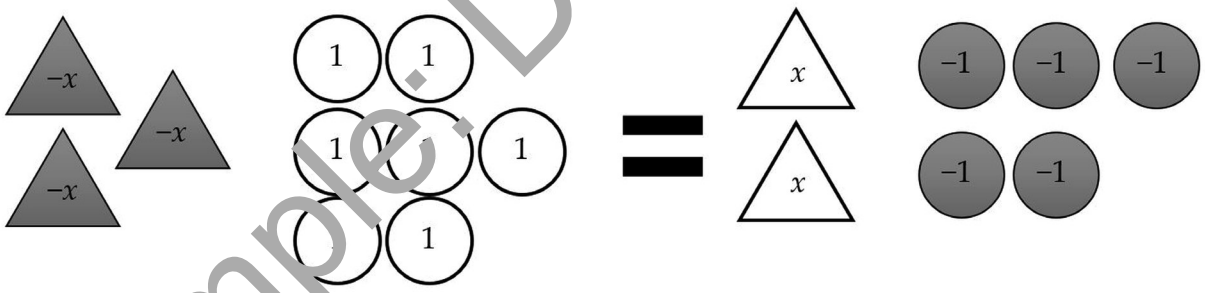
An **equation** is a relationship between two equivalent expressions. An equal sign (=) is used to indicate that the expression on the left has the same value as the expression on the right. If the expressions contain one variable, you can use the properties of algebra to solve the equation for the value of the variable.

**USEFUL PROPERTIES OF ALGEBRA**

| | |
|---|---|
| Additive Inverse: | $a + (-a) = 0$ |
| Multiplicative Inverse: | $a \div a = 1$ |
| | $a \times \frac{1}{a} = 1$ |
| Combine Like Terms: | $ax + bx = (a + b)x$ |
| Distributive Property: | $a(b + c) = ab + ac$ |

**NOTE** $a$, $b$, $c$, and $x$ represent real numbers.

You can solve an equation using one of many different strategies. One strategy is to use models to represent the equation. Then, you can use the properties of algebra to manipulate the model until you determine the value of the object that represents the variable. For example, you can use models to represent the equation $-3x + 7 = 2x - 5$. In this model, a triangle represents the variable, $x$, and a circle represents the unit constant, 1. Shaded triangles or circles indicate a negative value.




If the equation is presented or can be written symbolically, then you can use the properties of algebra to manipulate the constants and variable so that you can solve for the value of the variable

## EXAMPLES

**EXAMPLE 1:** What value of $x$ makes the equation $\frac{x}{5} - 18 = 2x + 27$ true?

**STEP 1** Isolate $x$ on one side of the equal sign and a real number on the other. Use inverse operations to manipulate both sides of the equation. Apply the additive inverse to add 18 to both sides of the equation.

$$\frac{x}{5} - 18 = 2x + 27$$
$$+18 \qquad +18$$
$$\frac{x}{5} = 2x + 45$$

**STEP 2** The variable, $x$, is being divided by 5. The inverse of division is multiplication, so apply the multiplicative inverse to multiply both sides of the equation by 5.

$$\frac{x}{5} = 2x + 45$$
$$5\left(\frac{x}{5}\right) = 5(2x + 45)$$
$$x = 10x + 225$$

**STEP 3** Apply the additive inverse to subtract $10x$ from both sides of the equation.

$$x = 10x + 225$$
$$-10x \quad -10x$$
$$-9x = 225$$

**$-9x = 225$**

**STEP 4** Apply the multiplicative inverse to divide both sides of the equation by −9.

$$\frac{-9x}{-9} = \frac{225}{-9}$$
$$x = -25$$

**$x = -25$**

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**EXAMPLE 2:** The measures of two angles are $(3.5x + 10)°$ and $(7x - 11)°$. What is the value of $x$ if the angles are congruent?

**STEP 1** If two angles are congruent, then their measures are equal. Write the equation showing the measures of the two angles are equivalent.

**$3.5x + 10 = 7x - 11$**

**STEP 2** Apply the additive inverse to subtract 7x from both sides of the equation.

$$3.5x + 10 = 7x - 11$$
$$-7x \qquad -7x$$
$$-3.5x + 10 = -11$$

**$-3.5x + 10 = -11$**

**STEP 3** Apply the additive inverse to subtract 10 from both sides of the equation.

$$-3.5x + 10 = -11$$
$$-10 \quad -10$$
$$-3.5x = -21$$

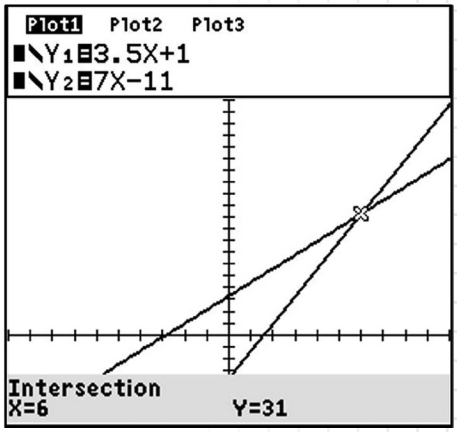
**$-3.5x = -21$**

**STEP 4** Apply the multiplicative inverse to divide both sides of the equation by −3.5.

$$\frac{-3.5x}{-3.5} = \frac{-21}{-3.5}$$
$$x = 6$$

**$x = 6$**

**STEP 5** Use graphing technology to verify your answer. Place the expression from the left side of the original equation into Y1 of the function editor and the expression from the right side into Y2 of the function editor. View the graph and calculate the intersection point. The $x$-coordinate of the intersection point is the solution to the original equation.




**YOU TRY IT!**

Marvelous Movies charges $5.50 for each soda and $5.00 for bottomless popcorn. Serious Cinema charges $2.50 for each soda and $14.00 for bottomless popcorn. How many sodas must be purchased in order for the total cost at each theater to be the same?

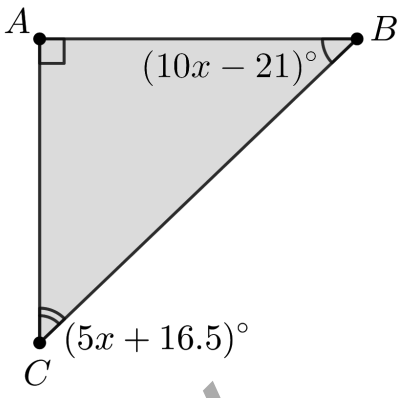
Let $x$ = _____

| Marvelous Movies | | Serious Cinema | |
|---|---|---|---|
| _____$x$ + | _____ | = _____$x$ + | _____ |
| Soda | Popcorn | Soda | Popcorn |

Solve for $x$.

$x$ = _____

**EXAMPLE 3:** The two acute interior angles of a right triangle are complementary. For $\Delta ABC$, what is the value of $x$? Record your answers and fill in the bubbles. Be sure to use the correct place value.




**STEP 1** If two angles are complementary, then their measures will add up to 90°. Write the equation.

- $m\angle B = (10x - 21)^\circ$ and $m\angle C = (5x + 16.5)^\circ$
- $m\angle B + m\angle C = 90^\circ$, so $m\angle B = 90^\circ - m\angle C$
- $(10x - 21)^\circ = 90^\circ - (5x + 16.5)^\circ$

**$10x - 21 = 90 - (5x + 16.5)$**

**STEP 2** Distribute the subtraction on the right side of the equation.

**$10x - 21 = 90 - 5x - 16.5$**

$10x - 21 = 90 - (5x + 16.5)$

$10x - 21 = 90 - 5x - 16.5$

**STEP 3** Apply the additive inverse to add $5x$ to both sides of the equation.

**$15x - 21 = 90 - 16.5$**

$10x - 21 = 90 - 5x - 16.5$

$+5x \qquad +5x$

$15x - 21 = 90 - 16.5$

**STEP 4** Combine like terms on the right hand side. Then, apply the additive inverse to add 21 to both sides of the equation.

**$15x = 94.5$**

$15x - 21 = 90 - 16.5$

$15x - 21 = 73.5$

$+21 \quad +21$

$15x = 94.5$

**STEP 5** Apply the multiplicative inverse to divide both sides of the equation by 15.

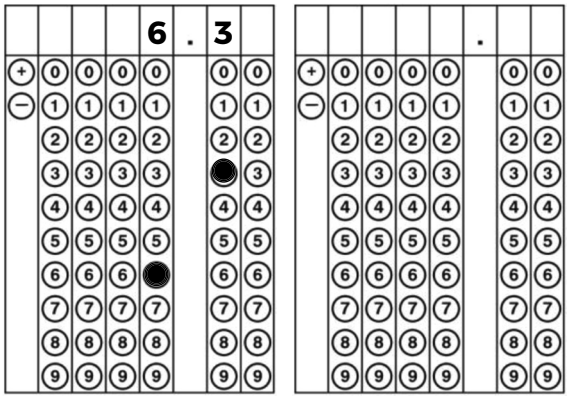
**$x = 6.3$**

$\frac{15x}{15} = \frac{94.5}{15}$

$x = 6.3$

**STEP 6** Since the question is a gridded response question, enter your response on the grid provided. Practice using the grid with the instructions.

1. Record a 6 in the ones column.
   Record a 3 in the tenths column.
   Ignore the sign since the answer is a positive number.
2. Bubble beneath the numeral 6.
   Bubble **3** beneath the numeral 3.



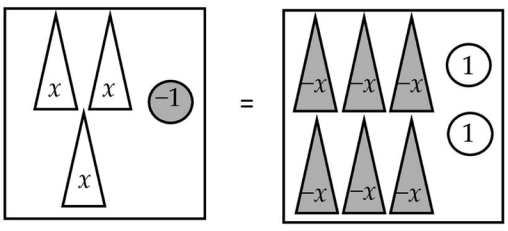

## PRACTICE

*For questions 1-3, determine the value of x that makes the equation true.*

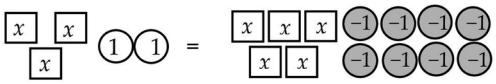
**1.** $6x - 7 = 8x + 23$

**2.** $\frac{x}{5} - 4 = 2 - \frac{2x}{5}$

**3.**



**4.** The equation $3x + 2 = 5x - 8$ is modeled below.

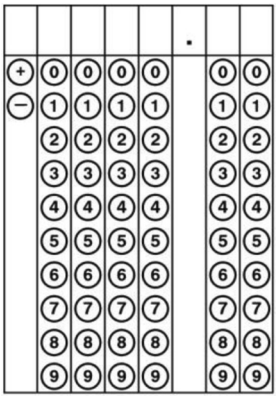


What value of $x$ makes the equation true?

**5.** One angle in a triangle measures $(6x + 54)°$. A second angle measures $(108 - 12x)°$. If the two angles are congruent, what is the measure of each angle?

**6.** Jasmine is planning a ski trip in the mountains with her friends. She can rent skis at the resort for $30 per day or she can buy skis to take with her for $150. Jasmine will also have to budget for a lift pass that costs $15 per day. How many days does she need to plan to ski in order for the cost of renting versus buying skis to cost the same for the trip?

**7.** A rectangle has a perimeter and area that are numerically equivalent. The length of the rectangle is 4.5 inches. What is the width of the rectangle? Record your answer and fill in the bubbles. Be sure to use the correct place value.



**8.** What value of x makes this equation true?

$$\frac{x}{3} + 1 = \frac{5x}{6} - 3$$

**A** $-8$

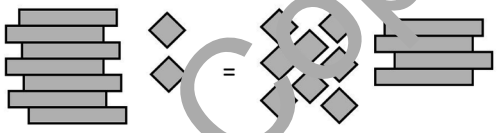
**B** 2

**C** 8

**D** $-3$

**9.** The model represents the equation

$$-7x - 2 = -8 - 4x.$$



What value of $x$ makes the equation true?

**F** 2

**G** $-\frac{4}{3}$

**H** $-2$

**J** $\frac{9}{11}$

**10.** In an isosceles triangle the measures of the two congruent angles are $(6x + 22)°$ and $(10x - 6)°$. If the measures of all 3 angles sum to 180°, what is the measure of the third angle in the triangle?

**A** 64°

**B** 52°

**C** 7°

**D** 166°