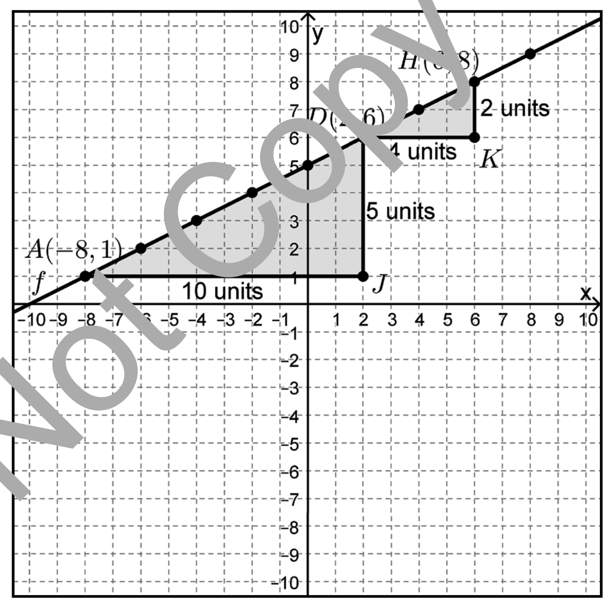
SLOPE AS A RATE OF CHANGE

8.4A

The student is expected to use similar right triangles to develop an understanding that slope, m, given as the rate comparing the change in y-values to the change in x-values, $\frac{y_2-y_1}{x_2-x_1}$, is the same for any two points (x_1, y_1) and (x_2, y_2) on the same line.

TELL ME MORE...

The slope of a line is a measure of the steepness of its graph. In the graph shown, line f has a slope of $\frac{1}{2}$, meaning that the ratio of the change in the vertical direction to the change in the horizontal direction is $\frac{1}{2}$.



Consider three points along line f: A (−8, 1), D (2, 6), and H (6, 8). Let $\overline{AD}$ be the hypotenuse of right triangle AJD and $\overline{DH}$ be the hypotenuse of right triangle DKH as shown in the graph.

- The vertical distance between points A and D is represented by $\overline{JD}$, which has a length equal to the difference of the y-coordinates of D and A: $6 - 1 = 5$.
- The horizontal distance between points A and D is represented by $\overline{AJ}$, which has a length equal to the difference of the x-coordinates of D and A: $2 - (-8) = 10$.
- The vertical distance between points D and H is represented by $\overline{KH}$, which has a length equal to the difference of the y-coordinates of H and D: $8 - 6 = 2$.
- The horizontal distance between points D and H is represented by $\overline{DK}$, which has a length equal to the difference of the x-coordinates of H and D: $6 - 2 = 4$.

Triangles AJD and DKH are similar, so the ratios of corresponding side lengths are equal.

$$\frac{JD}{AJ} = \frac{KH}{DK}$$

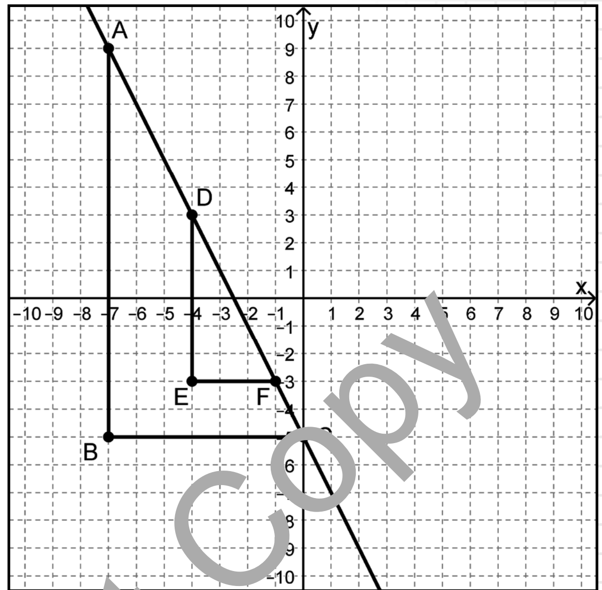
$$\frac{6-1}{2-(-8)} = \frac{8-6}{6-2}$$

$$\frac{5}{10} = \frac{2}{4}$$

The equivalent ratios represent the slope of line f. For any two points on line f, (x_1, y_1) and (x_2, y_2), the slope of line f is the ratio of the vertical distance $(y_2 - y_1)$ to the horizontal distance $(x_2 - x_1)$ between the two points.

EXAMPLES

EXAMPLE 1: Triangles *ABC* and *DEF* are similar right triangles. Write a proportion that could be used to show that the slope of $\overline{DF}$ is the same as the slope of $\overline{AC}$.



STEP 1 Identify the coordinates of points *A*, *C*, *D*, and *F*.

A (−7, 9)
C (0, −5)
D (−4, 3)
F (−1, −3)

STEP 2 Use the coordinates of *D* and *F* to write the slope of $\overline{DF}$ as the ratio of the vertical distance to the horizontal distance. Let *D* (−4, 3) = (x_1, y_1) and *F* (−1, −3) = (x_2, y_2).

$$\frac{\text{vertical distance}}{\text{horizontal distance}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 3}{-1 - (-4)}$$

Slope of $\overline{DF}$ = $\frac{-3 - 3}{-1 - (-4)}$

STEP 3 Use the coordinates of *A* and *C* to write the slope of $\overline{AC}$ as the ratio of the vertical distance to the horizontal distance. Let *A* (−7, 9) = (x_1, y_1) and *C* (0, −5) = (x_2, y_2).

$$\frac{\text{vertical distance}}{\text{horizontal distance}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - 9}{0 - (-7)}$$

Slope of $\overline{AC}$ = $\frac{-5 - 9}{0 - (-7)}$

STEP 4 A proportion is two or more equivalent ratios. Write a proportion setting the ratios for each slope equal to each other.

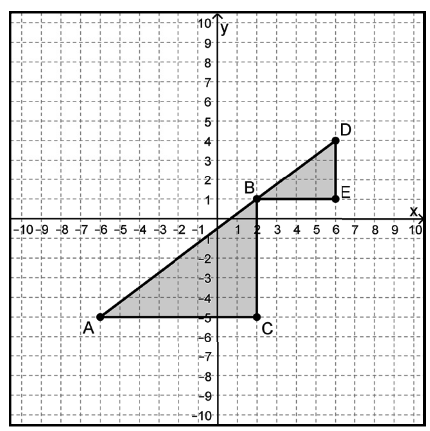
Slope of $\overline{DF}$ = Slope of $\overline{AC}$

$$\frac{-3 - 3}{-1 - (-4)} = \frac{-5 - 9}{0 - (-7)}$$

$$\frac{-6}{3} = \frac{-14}{7}$$

YOU TRY IT!

Triangle *ABC* and *BDE* are similar right triangles. Write a proportion using the coordinates of points *A*, *B*, and *D* to show that the slope of $\overline{AB}$ is equal to the slope of $\overline{BD}$.



Slope of $\overline{AB}$ = Slope of $\overline{BD}$

$$\frac{\square - \square}{\square - \square} = \frac{\square - \square}{\square - \square}$$

EXAMPLE 2: The table contains some points contained on line k. Triangle FGM is similar to triangle HJN. Write a proportion to show that the slope of $\overline{FG}$ is equal to the slope of $\overline{HJ}$.

	x	y
F	–8	6
G	–4	5
H	4	3
J	8	2

STEP 1 Use the coordinates of F and G to write the slope of $\overline{FG}$ as the ratio of the vertical distance to the horizontal distance. Let F (–8, 6) = (x_1, y_1) and G (–4, 5) = (x_2, y_2).

$$\frac{\text{vertical distance}}{\text{horizontal distance}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-6}{-4-(-8)}$$

Slope of $\overline{FG}$ = $\mathbf{\dfrac{5-6}{-4-(-8)}}$

STEP 2 Use the coordinates of H and J to write the slope of $\overline{HJ}$ as the ratio of the vertical distance to the horizontal distance. Let H (4, 3) = (x_1, y_1) and J (8, 2) = (x_2, y_2).

$$\frac{\text{vertical distance}}{\text{horizontal distance}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-3}{8-4}$$

Slope of $\overline{HJ}$ = $\mathbf{\dfrac{2-3}{8-4}}$

STEP 3 A proportion is two or more equivalent ratios. Write a proportion setting the ratios for each slope equal to each other.

Slope of $\overline{FG}$ = Slope of $\overline{HJ}$

$$\frac{5-6}{-4-(-8)} = \frac{2-3}{8-4}$$

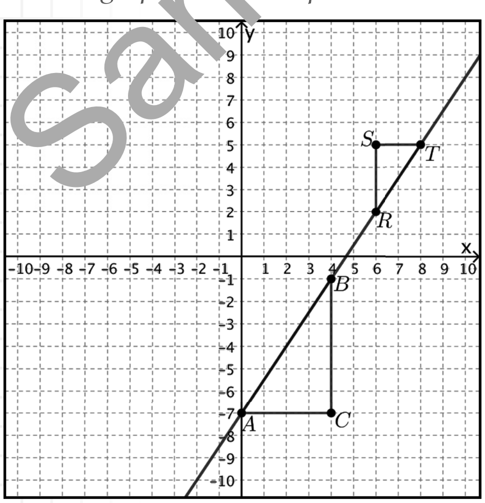
$$\frac{-1}{4} = \frac{-1}{4}$$

MAKE A NOTE . . .

Suppose that M (4, 8) and N (–2, 5). How does the slope of $\overline{MN}$ compare to the slope of $\overline{NM}$?

PRACTICE

Use the graph to answer questions 1 – 3.



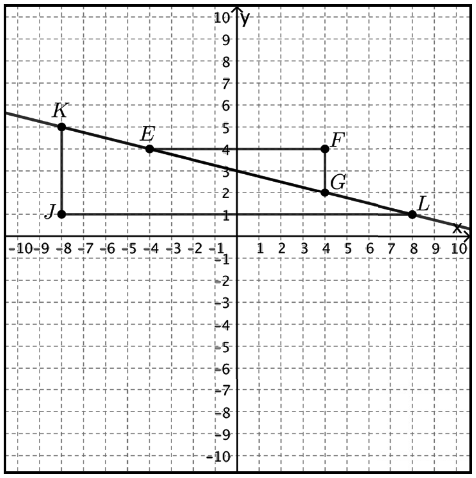
1. Write a ratio to show the slope of $\overline{AB}$.

2. Write a ratio to show the slope of $\overline{RT}$.

3. Complete the following statement using an inequality or equality symbol.

Slope of $\overline{AB}$ ______ Slope of $\overline{RT}$

Use the graph to answer questions 4 – 6.

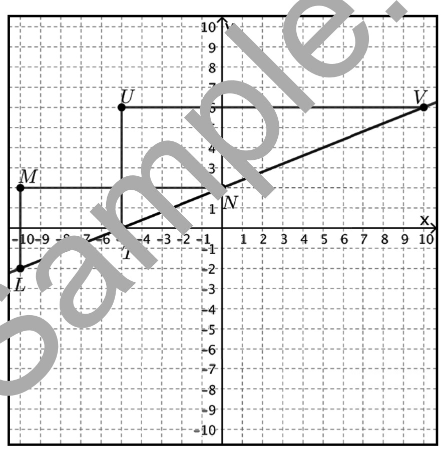


4. What ratio represents the slope of the hypotenuse of triangle *JKL*?

5. What ratio represents the slope of the hypotenuse of triangle *EFG*?

6. What is the relationship between the two slope values?

7. Triangles *LMN* and *TUV* are similar right triangles. Which proportion shows that the slope of $\overline{LN}$ and the slope of $\overline{TV}$ are equal?



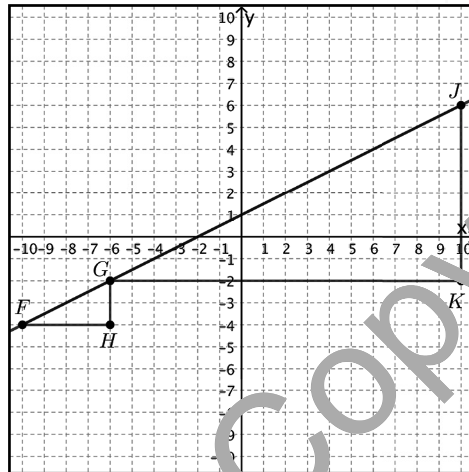
A $\frac{-2-2}{-10-0} = \frac{0-6}{-5-10}$

B $\frac{-10-2}{-2-0} = \frac{-5-6}{0-10}$

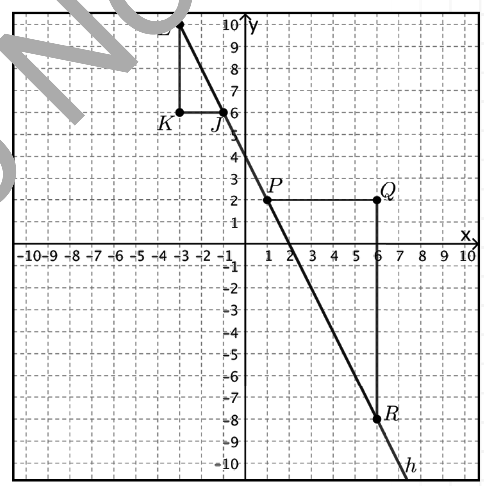
C $\frac{-10-0}{-2-2} = \frac{-5-10}{0-6}$

D $\frac{-2-0}{-10-2} = \frac{0-10}{-5-0}$

8. Triangle *FGH* and *GJK* are similar right triangles. Write a proportion using the coordinates of *F*, *G*, and *J* to show that the slopes of $\overline{FG}$ and $\overline{GJ}$ are equal.



9. Triangles *KJL* and *PQR* are similar right triangles plotted along line *h*.



Which statement is true?

F The slope of the hypotenuse of *JKL* is less than the slope of the hypotenuse of *PQR*.

G The slope of the hypotenuse of *JKL* is the same as the slope of the hypotenuse of *PQR*.

H The slope of the hypotenuse of *JKL* is greater than the slope of the hypotenuse of *PQR*.

J The slopes of the hypotenuses of *JKL* and *PQR* have no relationship.