

Cluster 8.3: Proportionality

8.3A: Similarity: Joselyn's Garden

Focusing TEKS

8.3A Proportionality. The student applies mathematical process standards to use proportional relationships to describe dilations. The student is expected to generalize that the ratio of corresponding sides of similar shapes are proportional, including a shape and its dilation. **Supporting Standard**

Additional TEKS:

8.3B Compare and contrast the attributes of a shape and its dilation(s) on a coordinate plane. **Supporting Standard**

7.5A Generalize the critical attributes of similarity, including ratios within and between similar figures. **Supporting Standard**

Focusing Mathematical Process

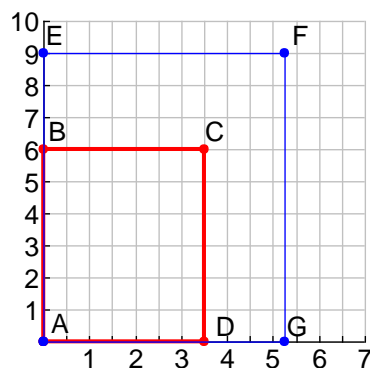
8.1B Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution.

8.1C Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems.

8.1F Analyze mathematical relationships to connect and communicate mathematical ideas.

▲ Performance Task

Joselyn is planning to expand her vegetable garden. She is making a sketch of the garden sizes for planning. The image below shows the two nested rectangular garden plots. The rectangle $ABCD$ is her current vegetable garden and the larger rectangle $AEFG$ represents her planned enlarged garden. Joselyn wants to ensure that her new garden is mathematically similar to her current one so she can enlarge the planting area the same for all vegetables. Is Joselyn's proposed enlarged garden area similar to her current garden area? Justify your reasoning.



Answer: Yes, rectangle $ABCD$ is mathematically similar to rectangle $AEFG$ because all pairs of corresponding sides are proportional and all corresponding angles are congruent. The scale factor applied to rectangle $ABCD$ to generate rectangle $AEFG$ is 1.5 or $\frac{3}{2}$.

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Mathematically Speaking...

In this task, students analyze two rectangles representing an original garden area and a planned enlargement shown plotted on a coordinate plane. Students are asked to determine if the shapes are mathematically similar. Once they have determined if the shapes are similar, students should express the similarity relationship using a scale factor, or ratio.

Students can use the ratio relationships between pairs of corresponding sides as one way to prove the shapes similar. Alternatively, they could analyze corresponding vertices of the rectangles and the scale factor relationship between the x- and y- values of the corresponding ordered pairs. If available, students can also use graphing software to plot the original rectangle, and dilate that original rectangle to create the enlarged garden area to verify their thinking.



This task builds upon the 7th grade skills students developed to generalize the attributes of similar figures using ratios.

Possible Solution

One way to know if shapes are similar is to write ratios comparing the lengths of the corresponding sides of the shapes in question, and to compare all corresponding pairs of angles. If all of the corresponding pairs of angles are congruent, and the ratios of the corresponding sides are equivalent; the shapes are considered mathematically similar.

There are 2 rectangles: $AEFG$ and $ABCD$. Since the figures are rectangles all of their angles are right angles. All right angles are congruent, $\angle A \cong \angle A$, $\angle E \cong \angle B$, $\angle F \cong \angle C$, $\angle G \cong \angle D$. To determine if the corresponding pairs of sides are proportional, write the ratios comparing the lengths of each pair of corresponding sides of rectangles $AEFG$ and $ABCD$.

Side AE corresponds to side AB . The ratio of the length of side AE to the length of side AB is

$$\frac{AE}{AB} = \frac{9}{6} = \frac{3}{2} \text{ in lowest terms.}$$

Side EF corresponds to side BC . The ratio of the length of side EF to the length of side BC is

$$\frac{EF}{BC} = \frac{5.25}{3.5} = \frac{3}{2} \text{ in lowest terms.}$$

Side FG corresponds to side CD . The ratio of the length of side FG to the length of side CD is

$$\frac{FG}{CD} = \frac{9}{6} = \frac{3}{2} \text{ in lowest terms.}$$

Side AG corresponds to side AD . The ratio of the length of side AG to the length of side AD is

$$\frac{AG}{AD} = \frac{5.25}{3.5} = \frac{3}{2} \text{ in lowest terms.}$$

Since all pairs of corresponding angles are congruent and all the ratios of corresponding sides of the rectangles are equivalent, the rectangles are mathematically similar.

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A scale factor is the value used to multiply by a linear dimension or coordinates of one figure to generate the linear dimensions or coordinates a similar figure. The scale factor applied to rectangle $ABCD$ to generate rectangle $AEFG$ is 1.5 or $\frac{3}{2}$. The scale factor applied to rectangle $AEFG$ to generate rectangle $ABCD$ is $\frac{2}{3}$.

Joselyn's enlarged garden space is mathematically similar to her current vegetable garden. It is an enlargement with a scale factor of $\frac{3}{2}$

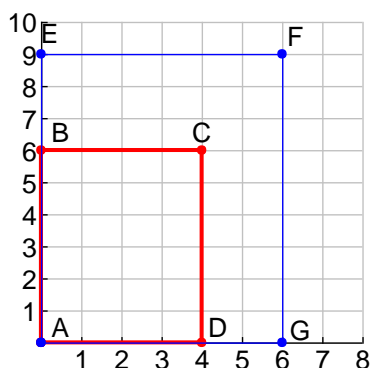
Look For...

- comparisons of corresponding angles and understanding that corresponding angles are congruent in similar figures
- comparisons of corresponding sides using ratios or proportions
- understand that to determine if figures are similar, all corresponding sides have a common proportional constant, known as the constant of proportionality
- understand a constant of proportionality > 1 means a figure is being enlarged, and a constant of proportionality < 1 means a figure is being reduced
- a common proportional constant of $\frac{3}{2}$ as the scale factor used to enlarge $ABCD$ to $AEFG$
- understanding of the definition of proportionally similar figures apparent in the processes used – corresponding angles are congruent and corresponding sides are proportional
- student justification of choices of solution strategy

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● Differentiation: Simplified Task

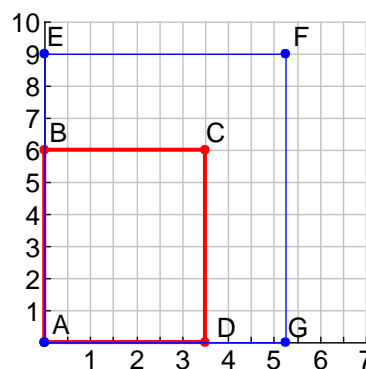
Joselyn is planning to expand her vegetable garden. She is making a sketch of the garden sizes for planning. The image below shows the two nested rectangular garden plots. The rectangle $ABCD$ is her current vegetable garden and the larger rectangle $AEFG$ represents her planned enlarged garden. Joselyn wants to ensure that her new garden is mathematically similar to her current one so she can enlarge the planting area the same for all vegetables. Is Joselyn's proposed enlarged garden area similar to her current garden area? Justify your reasoning.



Answer: Yes, rectangle $ABCD$ is mathematically similar to rectangle $AEFG$ because all pairs of corresponding sides are proportional and all corresponding angles are congruent. The scale factor applied to rectangle $ABCD$ to create rectangle $AEFG$ is 1.5 or $\frac{3}{2}$.

■ Differentiation: Enriching Task

Joselyn is planning to expand her vegetable garden. She is making a sketch of the garden sizes for planning. The image below shows the two nested rectangular garden plots. The rectangle $ABCD$ is her current vegetable garden and the larger rectangle $AEFG$ represents her planned enlarged garden. Joselyn wants to ensure that her new garden is mathematically similar to her current one so she can enlarge the planting area the same for all vegetables. Is Joselyn's proposed enlarged garden area similar to her current garden area? Justify your reasoning.



Joselyn decides to add a nested rectangular area for edible flowers in her vegetable garden. She sketches rectangle $AHIJ$ on her drawing by applying a scale factor of $\frac{3}{4}$ to rectangle $ABCD$. What will be the ordered pairs for each vertex of rectangle $AHIJ$?

Answer: Yes, rectangle $ABCD$ is mathematically similar to rectangle $AEFG$ because all pairs of corresponding sides are proportional and all corresponding angles are congruent. The scale factor applied to rectangle $ABCD$ to create rectangle $AEFG$ is 1.5 or $\frac{3}{2}$. The scale factor applied to rectangle $AEFG$ to create rectangle $ABCD$ is $\frac{2}{3}$.

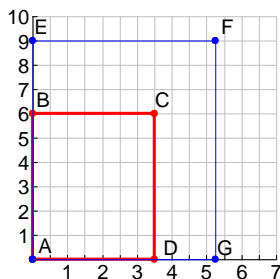
For rectangle $AHIJ$, $A(0, 0)$, $H(0, 4.5)$, $I(2.625, 4.5)$, $J(2.625, 0)$

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Scaffolded Task with Answers

Joselyn is planning to expand her vegetable garden. She is making a sketch of the garden sizes for planning. The image below shows the two nested rectangular garden plots. The rectangle $ABCD$ is her current vegetable garden and the larger rectangle $AEFG$ represents her planned enlarged garden. Joselyn wants to ensure that her new garden is mathematically similar to her current one so she can enlarge the planting area the same for all vegetables.



1. List pairs of corresponding angles between figure $ABCD$ and $AEFG$.

$\angle A$ and $\angle A$, $\angle E$ and $\angle B$, $\angle F$ and $\angle C$, $\angle G$ and $\angle D$

2. What do you know about each pair of corresponding angles?

All angles are right angles. All angle pairs are congruent. $\angle A \cong \angle A$, $\angle E \cong \angle B$, $\angle F \cong \angle C$, $\angle G \cong \angle D$

3. Write pairs of corresponding sides between figure $ABCD$ and $AEFG$ as fractions.

$$\frac{AE}{AB} = \frac{9}{6}; \frac{EF}{BC} = \frac{5.25}{3.5}; \frac{FG}{CD} = \frac{9}{6}; \frac{AG}{AD} = \frac{5.25}{3.5}$$

4. Reduce each of the four fractions above to lowest terms.

$$\frac{9}{6} = \frac{3}{2}; \frac{5.25}{3.5} = \frac{3}{2}; \frac{9}{6} = \frac{3}{2}; \frac{5.25}{3.5} = \frac{3}{2}$$

5. What do you know about each pair of corresponding sides?

Each pair of corresponding sides is a fraction equivalent to the ratio 3:2.

6. What is the definition of similar figures?

Corresponding angles are congruent and corresponding sides are proportional

7. What is the scale factor used to multiply the sides of $ABCD$ in order to create $AEFG$?

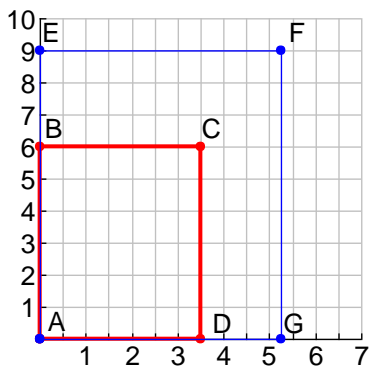
$$\frac{3}{2}$$

8. Is Joselyn's proposed enlarged garden area similar to her current garden area? Justify your reasoning.

Yes, because corresponding angles are congruent and corresponding sides are all proportional with a constant ratio of $\frac{3}{2}$.

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Procedural	0	1	2
Conceptual	0	1	2
Communication	0	1	2

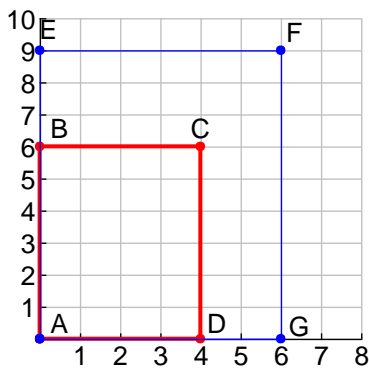
Total points: _____



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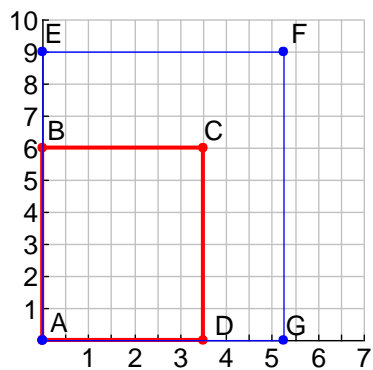
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Justify your reasoning.

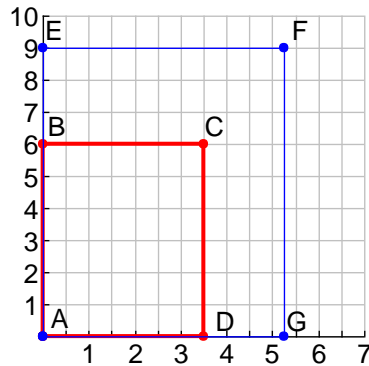
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1. List pairs of corresponding angles between figure $ABCD$ and $AEFG$.
2. What do you know about each pair of corresponding angles?
3. Write pairs of corresponding sides between figure $ABCD$ and $AEFG$ as fractions.
4. Reduce each of the four fractions above to lowest terms.
5. What do you know about each pair of corresponding sides?
6. What is the definition of similar figures?
7. What is the scale factor used to multiply the sides of $ABCD$ in order to create $AEFG$?
8. Is Joselyn's proposed enlarged garden area similar to her current garden area? Justify your reasoning.

