Cluster A.8: Quadratic Functions and Equations

A.8A: Solve Quadratic Functions using Factoring

Focusing TEKS

A.8A Quadratic Functions and Equations. The student applies the mathematical process standards to solve, with and without technology, quadratic equations and evaluate the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to solve quadratic equations having real solutions by factoring, taking square roots, completing the square, and applying the quadratic formula. **Readiness Standard**

Additional TEKS:

A.7A Graph quadratic functions on the coordinate plane, and use the graph to identify key attributes, if possible, including *x*-intercepts, *y*-intercept, zeros, maximum value, minimum value, vertex, and the equation of the axis of symmetry. **Readiness Standard**

A.7B Describe the relationships between the linear factors of quadratic expressions and the zeros of their associated quadratic functions. **Supporting Standard**

Focusing Mathematical Process

A.1A Apply mathematics to problems arising in everyday life, society, and the workplace.

A.1B Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem solving process and the reasonableness of the solution.

A.1D Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

A.1F Analyze mathematical relationships to connect and communicate mathematical ideas.

Performance Task

Charley is a golf pro and is analyzing his game to help him improve. Charley plotted the path of one of his golf balls on a coordinate plane using the function $f(x) = -16x^2 + 100x$ where x is the time in seconds and $f(x)$ is the height of the ball in feet. How long is this golf ball in the air between the time it was launched and the time it lands? How long did the ball take to reach its maximum height? Justify your reasoning.

***Answer**: The ball was in the air 6.25 seconds and reached its maximum height at 3.125 seconds.*

Cluster A.8: Quadratic Functions and Equations

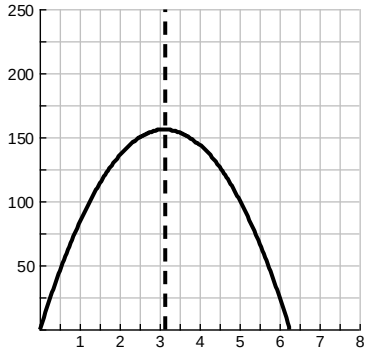
Mathematically Speaking...

In this task, students are being asked to evaluate a real-world situation that can be modeled using a quadratic function and to determine a method for solving the function equation in order to determine certain values that relate to the situation. Students will need to understand how the function rule can be used to represent the situation graphically and that the *x*-intercepts of the graph, or zeros/roots of the function, equate to the times the ball is on the ground. Students then also need to understand attributes of the graph of a quadratic function such as its vertex point and axis/line of symmetry to find the time the ball reached the maximum height.

While this problem easily lends itself to solving by factoring, students can use the graph of the function and the *x*-intercepts of the graph to find the related zeros for the height of the ball in order to solve for the travel time of the ball. Students can also use the axis of symmetry to find the time and height where the ball was at its maximum point, or could translate the function equation into vertex form to find this information.

Possible Solution

To find the time the ball was in the air you would need to determine the two times the ball was on the ground, the roots or zeros of the function which are also the *x*-intercepts of the graph of the function, and then the time is the amount of difference in the *x*-values.



First, factor the function rule. One way is to factor out the greatest common factor of -4*x*.

$$y = -16x^2 + 100x$$

$$y = (-4x)(4x - 25)$$

Then, set each of the two factors equal to zero to find the location of each of the times where the height of the ball was 0.

$(-4x) = 0$ $\quad$ $(4x - 25) = 0$

$x = 0$ $\quad$ $x = 6.25$

Since the difference between 6.25 and 0 is 6.25, the total time the ball was in the air is 6.25 seconds, the location of the second root of the function.

Next, you are asked to determine the time the ball reached its maximum height. The vertex of the function graph represents time and the maximum height of the ball. The x-value of this point is the time. One way to find the x-value of the vertex point is to finding the axis of symmetry for the graph of the function since the vertex lies on the axis of symmetry and the axis of symmetry cuts the graph exactly in half.

The equation for the axis of symmetry is $x = \frac{-b}{2a}$ where b is the number in front of the second term in a quadratic equation and a is the number in front of the first term in a quadratic equation. For the equation $y = -16x^2 + 100x$, the value of a is -16 and the value of b is 100. Substitute these numbers and use the equation for the axis of symmetry to solve for the x-value that represents the axis of symmetry.

$$x = \frac{-b}{2a} = \frac{-100}{2(-16)} = \frac{-100}{-32} = \frac{25}{8} = 3.125$$

Once you know the equation for the axis of symmetry, you also know the x-value in the vertex point since the vertex point lies on the axis of symmetry. In this case, the x-value of the vertex point is 3.125 seconds. Notice that this is also half of the total time the ball was in the air. Another way to find this number would have been to divide the total airtime of 6.25 seconds in half.

Differentiation: Simplified Task	Differentiation: Enriching Task
Charley is a golf pro and is analyzing his game to help him improve. Charley plotted the path of one of his golf balls on a coordinate plane using the function $f(x) = -16x^2 + 100x$ where x is the time in seconds and $f(x)$ is the height of the ball in feet. How long is this golf ball in the air between the time it was launched and the time it lands? Justify your reasoning. ***Answer:*** *The ball is in the air 6.25 seconds.*	Charley is a golf pro and is analyzing his game to help him improve. Charley plotted the path of one of his golf balls on a coordinate plane using the function $f(x) = -16x^2 + 100x$ where x is the time in seconds and $f(x)$ is the height of the ball in feet. How long is this golf ball in the air between the time it was launched and the time it lands? How long did the ball take to reach its maximum height? What is the maximum height the golf ball reached? Justify your reasoning. ***Answer:*** *The ball was in the air 6.25 seconds and reached its maximum height at 3.125 seconds. The maximum height of the ball was 156.25 feet.*

Name _____ Date _____

Performance Task: A.8A
Solve Quadratic Functions Using Factoring

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Procedural	0	1	2
Conceptual	0	1	2
Communication	0	1	2

Total points:_____

Name _____ Date _____

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Total points:_____

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Performance Task: A.8A

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Procedural	0	1	2
Conceptual	0	1	2
Communication	0	1	2

Total points:_____