Cluster 7.4: Proportionality

7.4A: Rates and Ratios: Hot Air Balloons

Focusing TEKS	Focusing Mathematical Process
7.4A Proportionality. The student applies mathematical process standards to represent and solve problems involving proportional relationships. The student is expected to represent constant rates of change in mathematical and real-world problems given pictorial, tabular, verbal, numeric, graphical, and algebraic representations, including $d = rt$. **Readiness Standard** **Additional TEKS:** 7.3A Add, subtract, multiply, and divide rational numbers fluently. **Supporting Standard** 7.4B Calculate unit rates from rates in mathematical and real-world problems. **Supporting Standard** 7.7A Represent linear relationships using verbal descriptions, tables, graphs, and equations that simplify to the form $y = mx + b$. **Readiness Standard** 6.6C Represent a given situation using verbal descriptions, tables, graphs, and equations in the form $y = kx$ or $y = x + b$. **Readiness Standard**	7.1A Apply mathematics to problems arising in everyday life, society, and the workplace. 7.1B Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. 7.1E Create and use representations to organize, record, and communicate mathematical ideas. 7.1F Analyze mathematical relationships to connect and communicate mathematical ideas.

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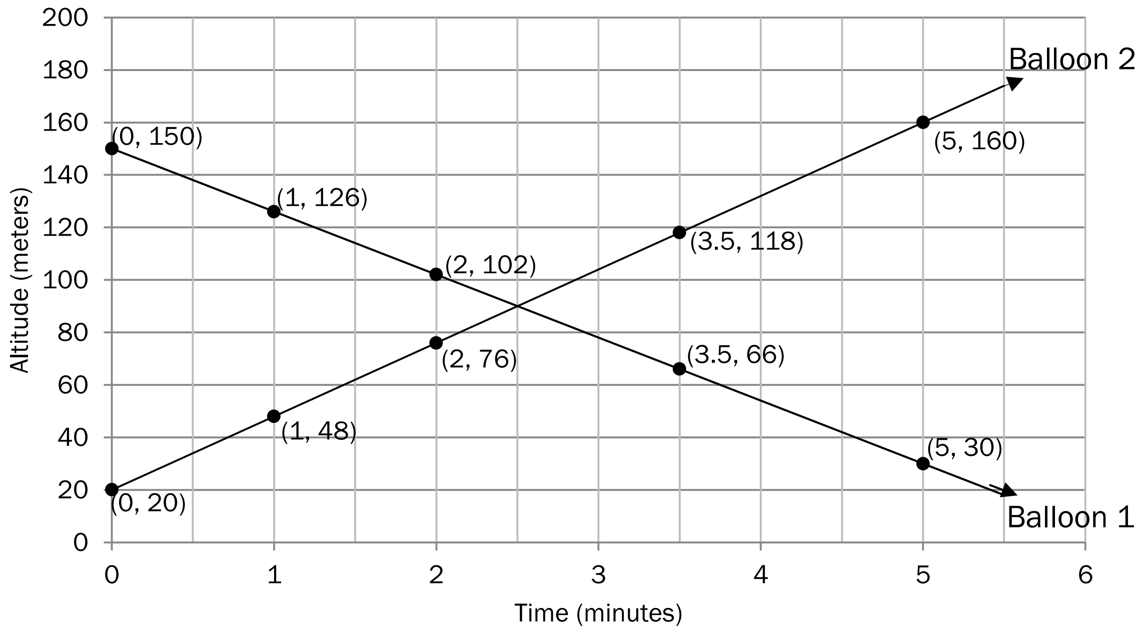
Performance Task

Walter and his family attended the local community hot air balloon festival last week. The data for two of the balloons Walter watched are shown in the graph below.



Which balloon has a faster rate of change in altitude? How much faster is this balloon moving? After approximately how many minutes will Balloon 1 land from the time Walter spotted the balloon? At what altitude will balloon 2 be at 7.5 minutes? Justify your reasoning.

Answer: *Balloon 2 is ascending at a faster rate than balloon 1 is descending, by 4 meters per minute. Balloon 2 is ascending at a rate of 30 m/min while balloon 1 is descending at a rate of 24 m/min. Balloon 1 will land about 6.25 minutes after Walter spotted it at 150 meters above the ground. Balloon 2 will be 230 meters off the ground at 7.5 minutes.*

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Cluster 7.4: Proportionality

Mathematically Speaking…

In this task, students are given a graph with 2 lines that represent the altitude over time of two hot air balloons seen at a festival. Students use the graph and the points on the lines to determine the rate of change in altitude for each balloon in order to decide which balloon has a greater rate of change. Once students have determined a rate of change for each balloon, they will use this information to determine specific values related to points on the line or that satisfy the equation of each line.

Students may use any appropriate method for calculating rate of change, including using the point on the graph, use of points in a table, equivalent ratios, or any other appropriate strategy. Students may also use any appropriate strategy to determine the specific values asked for the problem related to altitude or time at a specific point for each balloon.

The focus of the task is to ensure understanding of rate of change and how this information is applied in a real-world setting. This task builds upon student understanding of rates developed beginning in Grade 6 and incorporates applications of linear relationships.

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Cluster 7.4: Proportionality

Possible Solution

Balloon 1:

Balloon 1 is descending or coming down to land. The *y*-intercept value is the point where Walter spotted the balloon, which is (0, 150). The graph is marked on the *y*-axis in intervals of 4. Use a table to list some of the points marked on the line for balloon 1.

x	0	1	2	3.5	5
y	150	126	102	66	30

Use the ordered pairs to determine the rate of change. Using a pair of points, calculate the change in the *x*-values and the change in the *y*-value. Make a fraction of the change in *y* over the change in *x*. This fraction is the rate of change in altitude per the change in minutes. When simplifying to a rate of change in altitude over 1 minute, the result is the unit rate. Repeat this for all consecutive pairs of points to be sure the ratios in lowest terms are always the same.

+1 +1 +1.5 +1.5

x	0	1	2	3.5	5
y	150	126	102	66	30

−24 −24 −36 −36

In each pair of points, the ratio of *y* over *x* is always the same in lowest terms:

$$\frac{-24}{1} = \frac{-36}{1.5} = -24$$

Reasonableness

Note that the line plotted through the points on the graph is a straight line. A straight line is a linear function whose equation contains a constant rate of change. If a linear function exists, all pairs of coordinate points will have a constant ratio of difference in *y*-value over difference in *x*-value whenever the situation is linear.

The constant rate of change of −24 means that balloon 1 is descending at a rate of 24 meters per minute. This is the unit rate in the situation. The *y*-intercept is the starting point of the balloon from the time Walter spotted it at time 0. The equation in $y = mx + b$ form for balloon 1 is $y = -24x + 150$.

Balloon 2:

Balloon 1 is ascending or going up. The *y*-intercept value is the point where Walter spotted the balloon, which is (0, 20). The graph is marked on the *y*-axis in intervals of 4. Use a table to list some of the points marked on the line for balloon 1.

x	0	1	2	3.5	5
y	20	48	76	118	160

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Cluster 7.4: Proportionality

Use the ordered pairs to determine the rate of change. Using a pair of points, calculate the change in the *x*-values and the change in the *y*-value. Make a fraction of the change in *y* over the change in *x*. This fraction is the rate of change in altitude per the change in minutes. When simplifying to a rate of change in altitude over 1 minute, the result is the unit rate. Repeat this for all consecutive pairs of points to be sure the ratios in lowest terms are always the same.

+1 +1 +1.5 +1.5

x	0	1	2	3.5	5
y	20	48	76	118	160

+28 +28 +42 +42

In each pair of points, the ratio of *y* over *x* is always the same in lowest terms:

$$\frac{28}{1} = \frac{42}{1.5} = 28$$

The constant rate of change of 28 means that balloon 2 is ascending at a rate of 28 meters per minute. This is the unit rate in the situation. The *y*-intercept is the starting point of the balloon from the time Walter spotted it, at time 0. The equation in $y = mx + b$ form for balloon 2 is $y = 28x + 20$.

Balloon 1 is descending at a rate of 24 meters/minute and balloon 2 is ascending at a rate of 28 meters/minute. To determine which balloon is moving faster, use the absolute value of the rate in both equations. Since 28 is greater than 24, balloon 2 is moving faster by 4 meters per minute.

To find the approximate number of minutes it takes for balloon 1 to land, determine the value of *x* when the value of *y* is 0, representing a 0 altitude. Use substitution in the equation of the value 0 for *y* or use the graph to solve.

$$0 = -24x + 150$$
$$0 - 150 = -24x + 150 - 150$$
$$-150 = -24x$$
$$\frac{-150}{-24} = \frac{-24x}{-24}$$
$$6.25 = x$$

Balloon 1 will land after 6.25 minutes of Walter's spotting.

To find the altitude of balloon 2 at a time of 7.5 minutes after Walter spotted it, substitute 7.5 into the equation for balloon 2. Use the value 7.5 in place of *x* in the equation since the *x*-values represent time and the *y*-values represent altitude.

$$y = 28(7.5) + 20$$
$$y = 210 + 20$$
$$y = 230$$

Balloon 2 was at an altitude of 230 meters at 7.5 minutes after being spotted by Walter.

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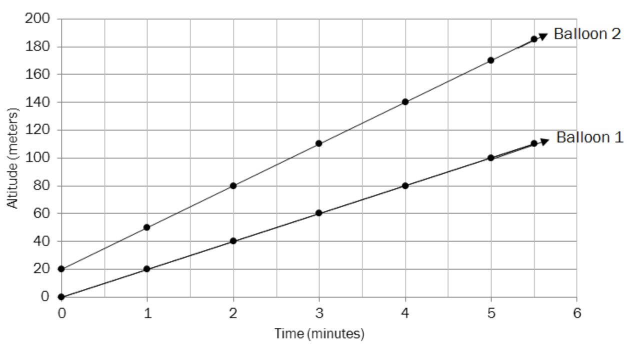
Cluster 7.4: Proportionality

Look For…

- proficiency with rational number operations
- a solution strategy to determine the rate of change in altitude for each balloon and to compare the rates
- correct computations of rate for each balloon, correct determination of faster rate, and correct values for the specific function values of altitude and time
- student justification of choices of solution strategy

Differentiation: Simplified Task

Walter and his family attended the local community hot air balloon festival last week. The data for two of the balloons Walter watched are shown in the graph below.

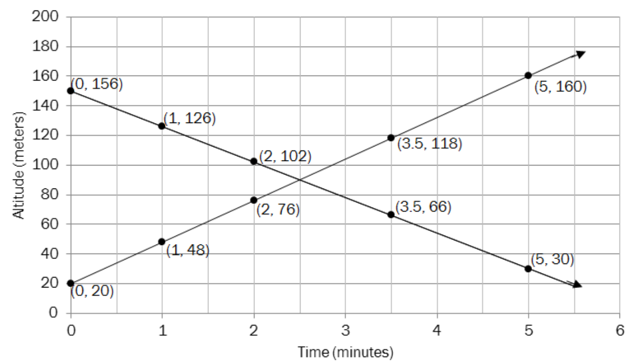


Which balloon has a faster rate of change in altitude? How much faster is this balloon moving? Justify your reasoning.

Answer: *Balloon 2 has a faster rate of change in altitude and is ascending at 10 meters/minute faster than Balloon 1.*

Differentiation: Enriching Task

Walter and his family attended the local community hot air balloon festival last week. The data for two of the balloons Walter watched are shown in the graph below.



Which balloon has a faster rate of change in altitude? How much faster is this balloon moving? After approximately how many minutes will Balloon 1 land from the time Walter spotted the balloon? What will be the altitude of balloon 2 be at 7.5 minutes? At about what time, to the nearest tenth, were the two hot air balloons at the same altitude? Justify your reasoning.

Answer: *Balloon 2 is ascending at a faster rate than balloon 1 is descending, by 6 meters per minute. Balloon 2 is ascending at a rate of 30 m/min while balloon 1 is descending at a rate of 24 m/min. Balloon 1 will land about 6.5 minutes after Walter spotted it at 156 meters above the ground. Balloon 2 will be 230 meters off the ground after 7.5 minutes. The balloons were at the same altitude about 2.5minutes.*

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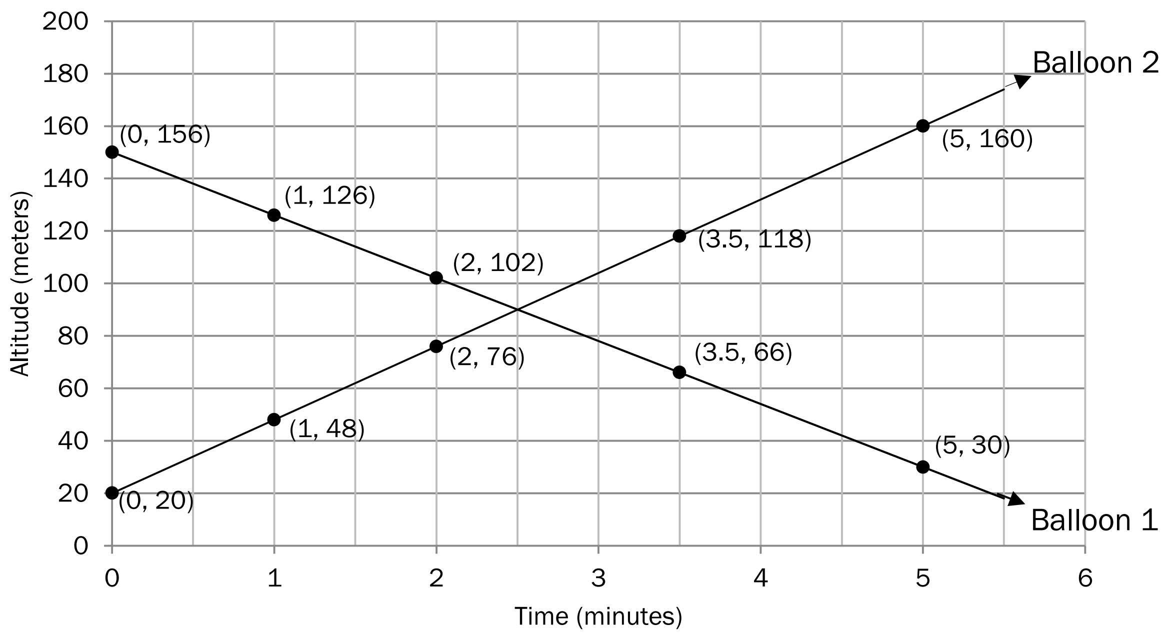
Cluster 7.4: Proportionality

Scaffolded Task with Answers

Walter and his family attended the local community hot air balloon festival last week. The data for two of the balloons Walter watched are shown in the graph below.



1. What is the *y*-intercept for balloon 1?
   (0, 150)
2. What is the rate of change in altitude for balloon 1?
   −24
3. What linear equation best represents balloon 1?
   $y = -24x + 150$
4. What is the *y*-intercept for balloon 2?
   (0, 20)
5. What is the rate of change in altitude for balloon 2?
   28
6. What linear equation best represents balloon 2?
   $y = 28x + 20$
7. Which balloon has a faster rate of change in altitude?
   Balloon 2
8. How much faster is this balloon moving?
   4 meters/minute

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9. After approximately how many minutes will Balloon 1 land from the time Walter spotted the balloon?
   About 6.25 minutes

10. At what altitude will balloon 2 be at 7.5 minutes?
    230 meters

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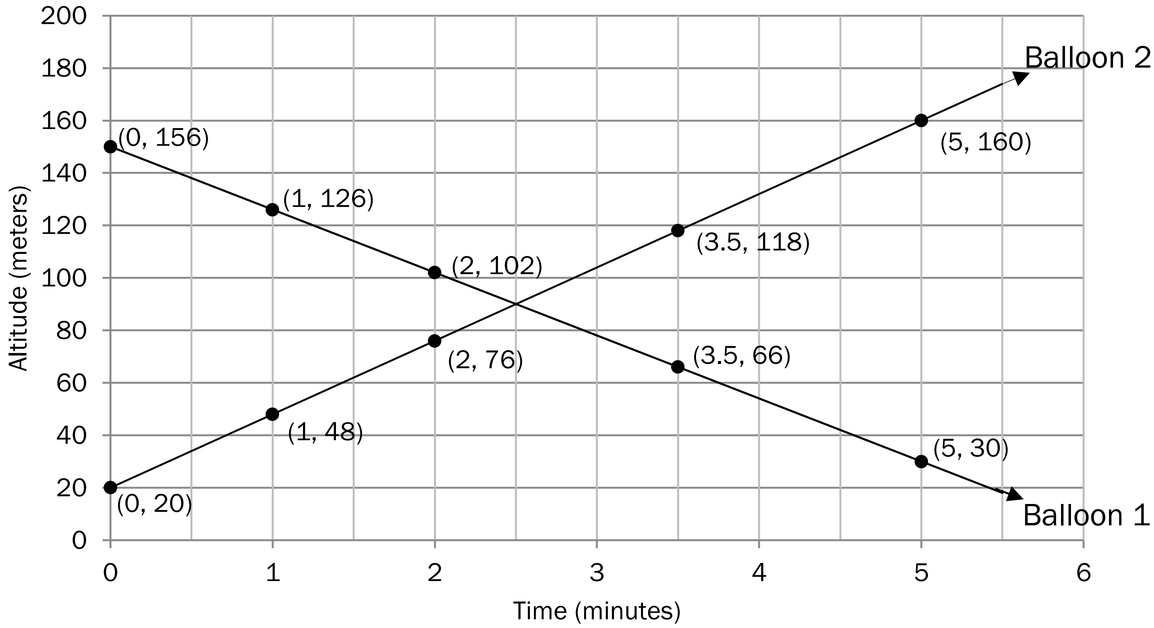
Performance Task: 7.4A

Rates and Ratios: Hot Air Balloons

Walter and his family attended the local community hot air balloon festival last week. The data for two of the balloons Walter watched are shown in the graph below.



- Which balloon has a faster rate of change in altitude?
- How much faster is this balloon moving?
- After approximately how many minutes will Balloon 1 land from the time Walter spotted the balloon?
- At what altitude will balloon 2 be at 7.5 minutes?

Justify your reasoning.

Procedural	0	1	2
Conceptual	0	1	2
Communication	0	1	2

Total points:_______

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Name ____ Date ___________________________

Performance Task: 7.4A

Rates and Ratios: Hot Air Balloons

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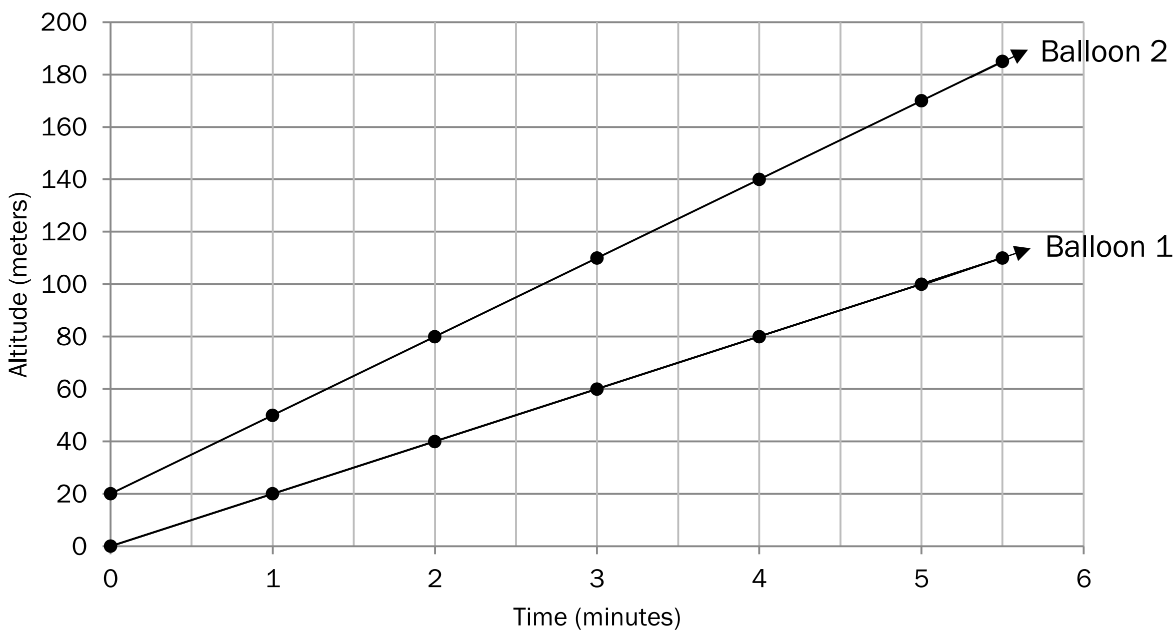

- Which balloon has a faster rate of change in altitude?
- How much faster is this balloon moving?

Justify your reasoning.

Procedural	0	1	2
Conceptual	0	1	2
Communication	0	1	2

Total points:_______

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Name ____ Date __________________________

Performance Task: 7.4A
Rates and Ratios: Hot Air Balloons

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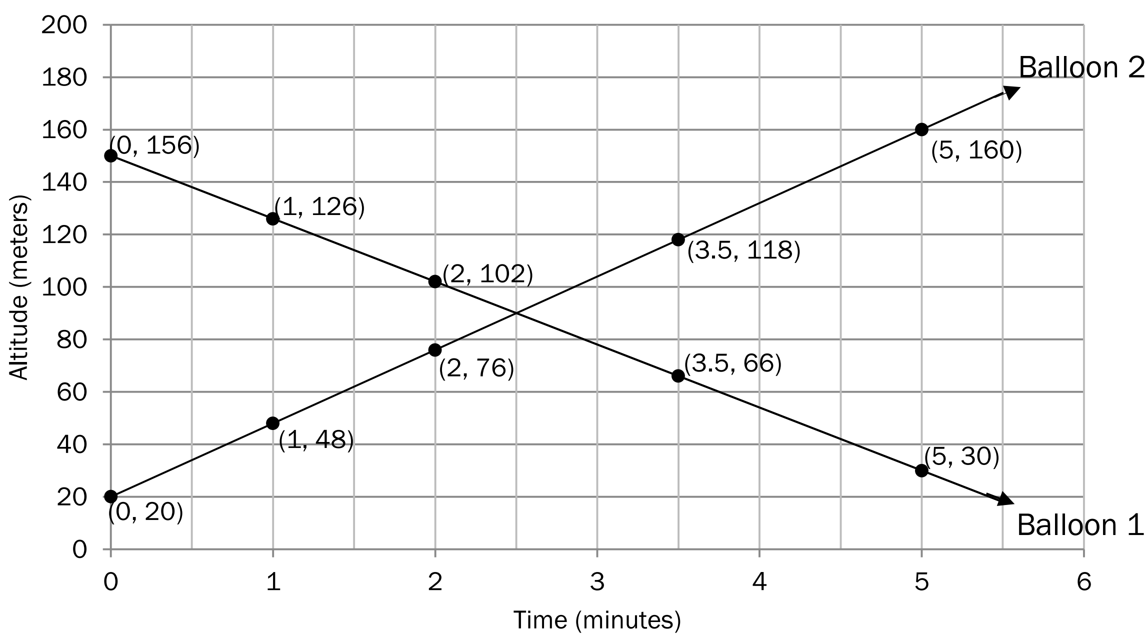

- Which balloon has a faster rate of change in altitude?
- How much faster is this balloon moving?
- After approximately how many minutes will Balloon 1 land from the time Walter spotted the balloon?
- What will be the altitude of balloon 2 be at 7.5 minutes?
- At about what time, to the nearest tenth` , were the two hot air balloons at the same altitude?

Justify your reasoning.

Procedural	0	1	2
Conceptual	0	1	2
Communication	0	1	2

Total points:_______

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Name ____ Date ___________________________

Performance Task: 7.4A

Rates and Ratios: Hot Air Balloons

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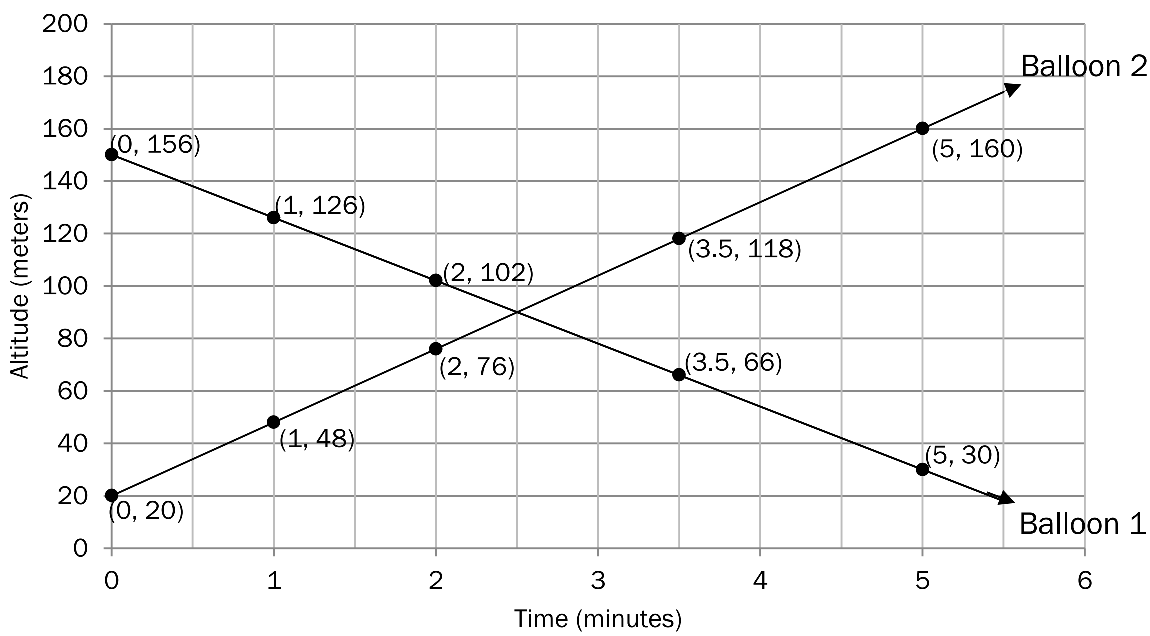

1. What is the y-intercept for balloon 1?

2. What is the rate of change in altitude for balloon 1?

3. What linear equation best represents balloon 1?

4. What is the y-intercept for balloon 2?

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Name ______________________________ Date ____________________

5. What is the rate of change in altitude for balloon 2?

6. What linear equation best represents balloon 2?

7. Which balloon has a faster rate of change in altitude?

8. How much faster is this balloon moving?

9. After approximately how many minutes will Balloon 1 land from the time Walter spotted the balloon?

10. At what altitude will balloon 2 be at 7.5 minutes?

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