New Courses, New Opportunities: Algebraic Reasoning

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#TXAlgReas

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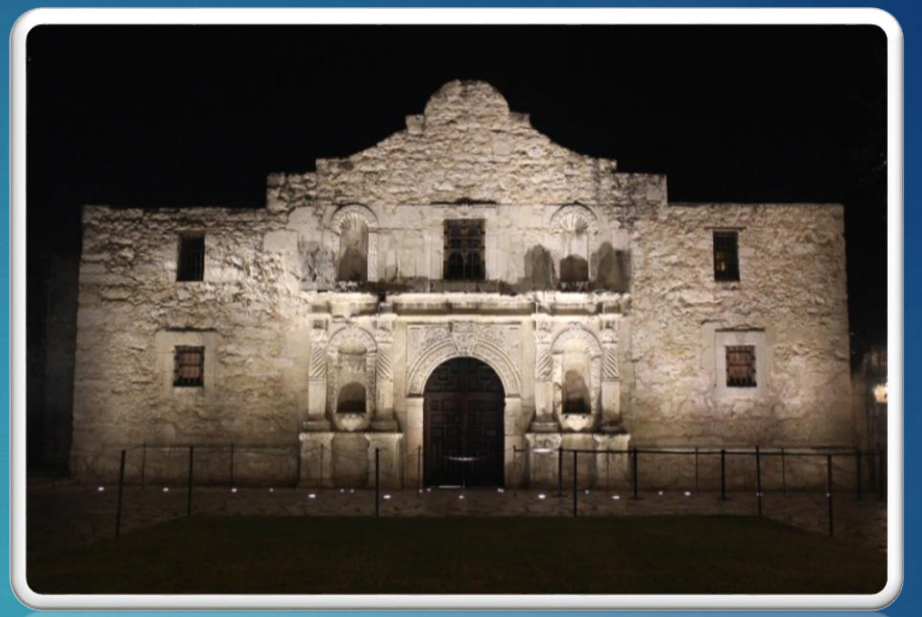
Overview

- HB5: Pathways and Possibilities
- Building a Graduation Pathway
- New Course: Algebraic Reasoning
- Effective Instruction



HB5: Pathways and Possibilities

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HB5 Graduation Requirements

Foundation

- 3 math credits
- Algebra 1 and Geometry required
- "Advanced Math"

Endorsement

- 4th math credit
- 5 possible endorsements
  - STEM
  - Business and Industry
  - Arts and Humanities
  - Public Service
  - Multidisciplinary
- STEM: Must include Algebra 2

HB5 – Foundation Requirements Advanced Mathematics

- Mathematical Models with Applications;
- Mathematical Applications in Agriculture, Food, and Natural Resources (CTE);
- Digital Electronics (CTE); and
- Robotics Programming and Design (Tech).

- Algebra II;
- Precalculus;
- Advanced Quantitative Reasoning;
- Independent Study in Mathematics;
- Discrete Mathematics for Problem Solving;
- Algebraic Reasoning;
- Statistics;
- AP Statistics;
- AP Calculus AB, AP Calculus BC
- AP Computer Science;
- IB Mathematics courses

- Engineering Mathematics; (CTE)
- Statistics and Risk Management; (CTE)

  New title: Statistics and Business Decision Making (2017)
- Discrete Mathematics for Computer Science; (Tech)

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HB5 – Endorsement Possibilities

To earn an endorsement, a student must obtain a fourth mathematics credit (1.0 or 0.5 + 0.5)

- Algebra II;
- Precalculus;
- Advanced Quantitative Reasoning;
- Independent Study in Mathematics;
- Discrete Mathematics for Problem Solving;
- Algebraic Reasoning;
- Statistics;
- AP Statistics, Calculus, Computer Science;
- International Baccalaureate (IB) courses
- Engineering Mathematics; (CTE)
- Statistics and Risk Management; (CTE)
- Discrete Mathematics for Computer Science; (Tech)
- IHE credit eligible courses
- Courses required for industry-recognized credentials
- College Preparatory Mathematics course

Beginning in 2017-18:

- Accounting II (CTE)
- Applied Math for Technical Professionals (CTE – Car. Dev.)
- Financial Mathematics (CTE)
- Manufacturing Engineering Technology II (CTE – Alg2 Recommended)
- Math for Medical Professionals (CTE)
- Robotics II (CTE)

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Building a Graduation Pathway

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Possible Pathway
STEM: Engineering Focus

Middle School	9th Grade	10th Grade	11th Grade	12th Grade
	Algebra 1 (F)	Geometry (F)	Algebra 2 (F)	Precalculus (E)
	ROTC/PE	Concepts of Engineering and Technology (E)	Principles of Technology (E)	Engineering Design and Problem Solving (E)
	English 1 (F)	English 2 (F)	English 3 (F)	English 4 (F)
	Foreign Language 1 (F)	Foreign Language 2 (F)	Fine Art (F)	Engineering Mathematics (E)
	Biology (F)	Chemistry (F)	Physics (F)	AP Physics, AP Biology, or AP Chemistry (E)
	World History (F)	World Geography (E)	US History (F)	Government/ Economics (F)
	Elective (F)	Elective (F)	Elective (F)	Elective (F)

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This pathway: Algebra 2 + 2 math credits (Precalculus and Engineering Mathematics)
Also, coherent CTE sequence 4+ courses

Possible Pathway
Public Services: Medical Focus

Middle School	9th Grade	10th Grade	11th Grade	12th Grade
	Algebra 1 (F)	Geometry (F)	Algebra 2 (F)	Mathematics for Medical Professionals* (E)
	ROTC/PE	Principles of Health Science (E)	Health Science Theory (E)	Practicum in Health Science I (E)
	English 1 (F)	English 2 (F)	English 3 (F)	English 4 (F)
	Foreign Language 1 (F)	Foreign Language 2 (F)	Fine Art (F)	Practicum in Health Science I (E) (2nd credit)
	Biology (F)	Chemistry (F)	Anatomy & Physiology (F)	Medical Microbiology (E)
	World History (F)	World Geography (E)	US History (F)	Government/ Economics (F)
	Elective (F)	Elective (F)	Elective (F)	Elective (F)

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Electives required for Public Services Endorsement (4+ coherent CTE, including #3+ in a sequence)

Possible Pathway
Multidisciplinary Studies

Middle School	9th Grade	10th Grade	11th Grade	12th Grade
	Algebra 1 (F)	Geometry (F)	Algebraic Reasoning (F)	Statistics* (E)
	ROTC/PE	Elective (E)	Elective (E)	Elective (E)
	English 1 (F)	English 2 (F)	English 3 (F)	English 4 (F/E)
	Foreign Language 1 (F)	Foreign Language 2 (F)	Fine Art (F)	Elective
	Biology (F)	Chemistry (F)	Physics(F)	Earth Space Science (E)
	World History (F)	World Geography (E)	US History (F)	Government/ Economics (F)
	Elective (F)	Elective (F)	Elective (F)	Elective (F)

- four credits in each of the four foundation subject areas to include English IV and chemistry and/or physics
- 2 additional elective credits

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New Course: Algebraic Reasoning

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Algebraic Reasoning: Background

- Impetus: Not enough "advanced math" that didn't require Algebra 2 as a prerequisite
- Originally envisioned by Texas SBOE as an "applied Algebra 2" course
- Originally thought of as an Algebra 2 alternative
- Committee was assembled in 2014 to develop TEKS for both Algebraic Reasoning and non-AP Statistics
- TEKS were approved in 2015 for implementation in the 2015-16 school year.
- Instructional materials to be reviewed under Proclamation 2017

Algebraic Reasoning: TEKS

- AR.1: Mathematical Processes
- AR.2: Patterns and Structure
  - Finite differences and functions
- AR.3: Patterns and Structure
  - Attributes of Algebra 2 function families
- AR.4: Number and Algebraic Methods
  - Function operations
- AR.5: Number and Algebraic Methods
  - Matrices
- AR.6: Number and Algebraic Methods
  - Estimating and determining solutions to equations related to functions
- AR.7: Modeling from Data
  - Using functions to model real-world data

Supports for Algebra 2	Algebraic Reasoning	Algebra 2
Attributes of Functions	Compare and contrast key attributes of functions ($f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$, $f(x) = {}^1/_x$, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = \lvert x \rvert$, and $f(x) = \log_b(x)$ where b is 10 or e) tabularly, graphically, and symbolically (AR.3A).	Graph and analyze key attributes of graph the functions $f(x)=\sqrt{x}$, $f(x)=1/x$, $f(x)=x^3$, $f(x)= 3\sqrt{x}$, $f(x)=bx$, $f(x)=\lvert x \rvert$, and $f(x)=\log b(x)$ where b is 2, 10, and e (2A.2A)
Systems of Equations	Use matrices and technology to solve systems of 3 linear equations (AR.5E)	Use matrices, substitution, elimination, and technology to solve systems of 3 linear equations (2A.3B)
Solving Equations	Approximate the solutions to equations related to exponential, logarithmic, square root, and cubic functions (AR.6C)	Solve exponential and logarithmic equations (2A.5D), square root equations (2A.4F), and cube root equations (2A.6B)
Modeling with Functions	Use finite differences and average rates of change to determine function models (linear, quadratic, and exponential) for real-world data sets (AR.7D) and use them to solve problems (AR.7C).	Use regression methods through technology to determine function models (linear, quadratic, exponential) for data sets (2A.8B) and use them to solve problems (2A.8C).

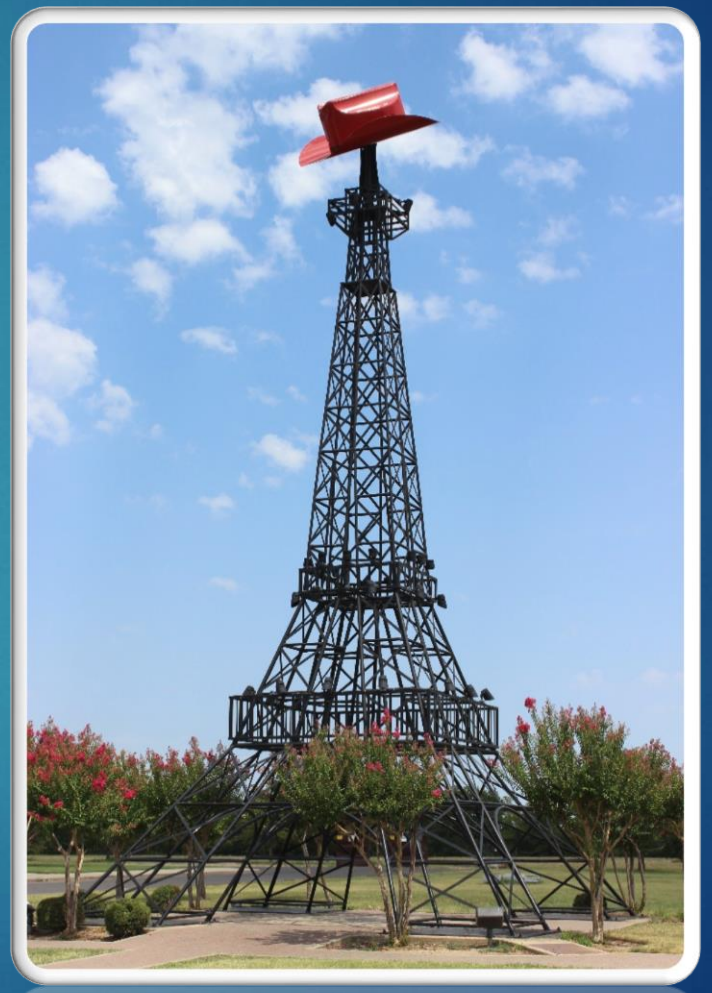
How is Algebraic Reasoning Being Used?

- Algebra 1 is a required prerequisite. Otherwise…

Algebra 1	Algebraic Reasoning	Geometry	Algebra 2
Algebra 1	Geometry	Algebraic Reasoning	Algebra 2
Algebra 1	Math Models	Geometry	Algebraic Reasoning
Algebra 1	Geometry	Algebraic Reasoning	Statistics
Algebra 1	Geometry	Algebraic Reasoning	CTE Math
Algebra 1	Geometry	Algebraic Reasoning	Financial Math

No one ever says to a teacher, "Wow, I remember that really cool worksheet we did!"

--Karen Karp, CAMT 2016 Opening Session



Effective Instruction

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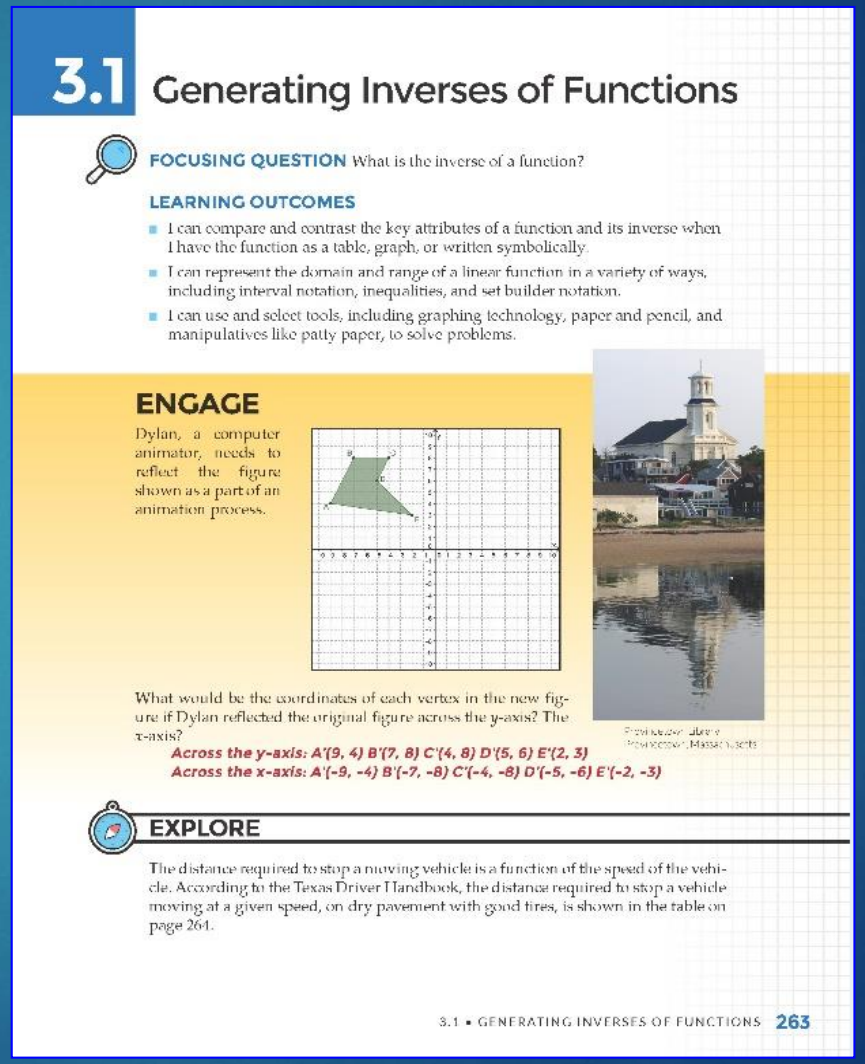
Inquiry-Based Instruction: 5E

- Engage: connect back to prior knowledge.
- In this lesson, the Engage reaches back to reflections with coordinate geometry (Grade 8, Geometry)
- Explore: investigation using technology, data, or concrete objects.



3.1 Generating Inverses of Functions

FOCUSING QUESTION What is the inverse of a function?

LEARNING OUTCOMES

- I can compare and contrast the key attributes of a function and its inverse when I have the function as a table, graph, or written symbolically.
- I can represent the domain and range of a linear function in a variety of ways, including interval notation, inequalities, and set builder notation.
- I can use and select tools, including graphing technology, paper and pencil, and manipulatives like patty paper, to solve problems.

ENGAGE

Dylan, a computer animator, needs to reflect the figure shown as a part of an animation process.

What would be the coordinates of each vertex in the new figure if Dylan reflected the original figure across the y-axis? The x-axis?

Across the y-axis: A'(9, 4) B'(7, 8) C'(4, 8) D'(5, 6) E'(2, 3)
Across the x-axis: A'(-9, -4) B'(-7, -8) C'(-4, -8) D'(-5, -6) E'(-2, -3)

EXPLORE

The distance required to stop a moving vehicle is a function of the speed of the vehicle. According to the Texas Driver Handbook, the distance required to stop a vehicle moving at a given speed, on dry pavement with good tires, is shown in the table on page 264.

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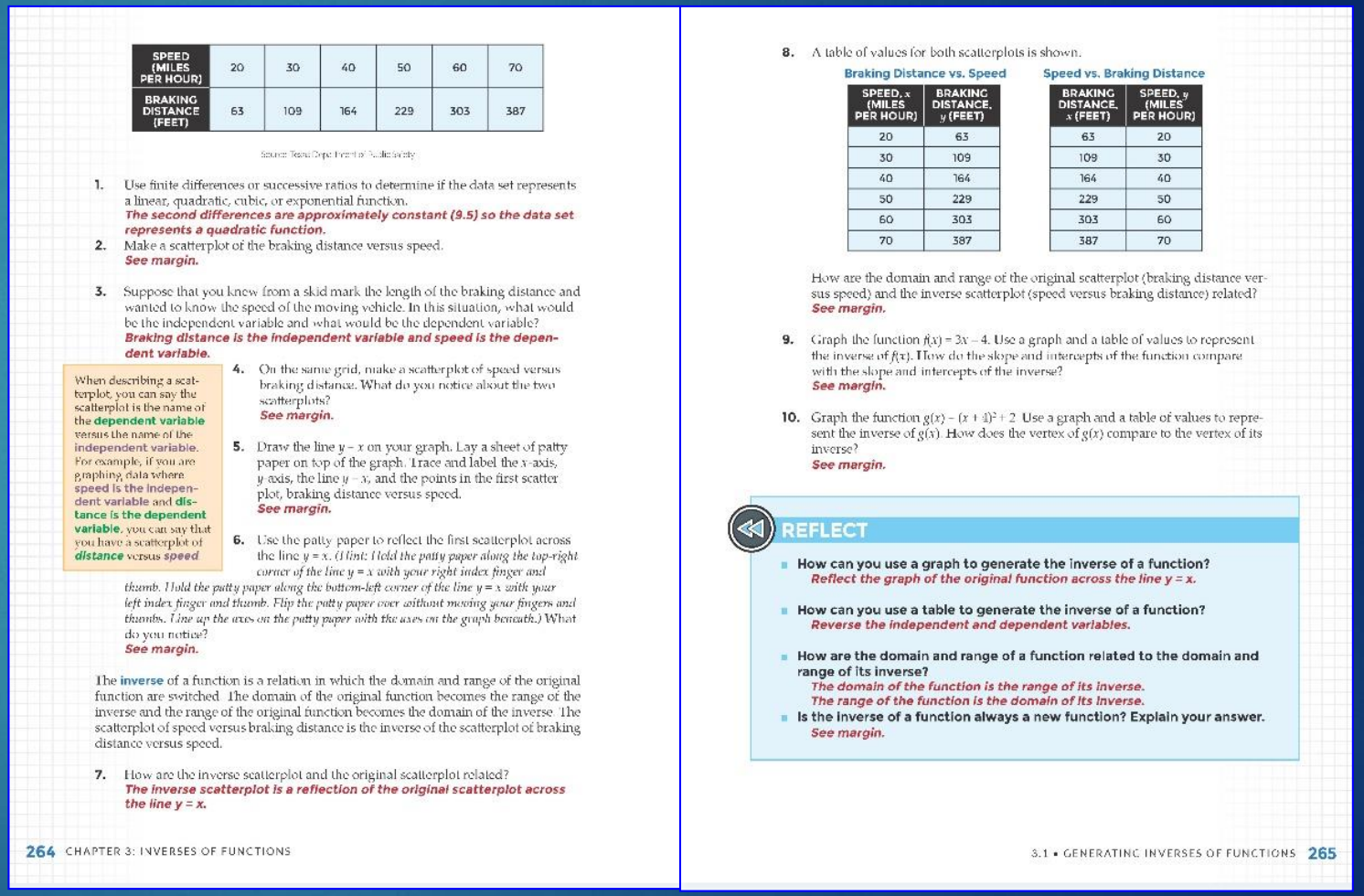
Inquiry-Based Instruction: 5E

- Explore: In this case, real-world data is used to generate a scatterplot.
- Patty paper is used to reflect the data across the line y = x (geometric view of an inverse)
- Tables are used to organize the data.
- Reflective questions are used to begin formalizing key ideas.



SPEED (MILES PER HOUR)	20	30	40	50	60	70
BRAKING DISTANCE (FEET)	63	109	164	229	303	387

1. Use finite differences or successive ratios to determine if the data set represents a linear, quadratic, cubic, or exponential function.
The second differences are approximately constant (9.5) so the data set represents a quadratic function.
2. Make a scatterplot of the braking distance versus speed.
See margin.
3. Suppose that you knew from a skid mark the length of the braking distance and wanted to know the speed of the moving vehicle. In this situation, what would be the independent variable and what would be the dependent variable?
Braking distance is the independent variable and speed is the dependent variable.

When describing a scatterplot, you can say the scatterplot is the name of the **dependent variable** versus the name of the **independent variable**. For example, if you are graphing data where **speed is the independent variable** and **distance is the dependent variable**, you can say that you have a scatterplot of ***distance*** versus ***speed***

4. On the same grid, make a scatterplot of speed versus braking distance. What do you notice about the two scatterplots?
See margin.
5. Draw the line $y = x$ on your graph. Lay a sheet of patty paper on top of the graph. Trace and label the x-axis, y-axis, the line $y = x$, and the points in the first scatter plot, braking distance versus speed.
See margin.
6. Use the patty paper to reflect the first scatterplot across the line $y = x$. *(Hint: Hold the patty paper along the top-right corner of the line $y = x$ with your right index finger and thumb. Hold the patty paper along the bottom-left corner of the line $y = x$ with your left index finger and thumb. Flip the patty paper over without moving your fingers and thumbs. Line up the axes on the patty paper with the axes on the graph beneath.)* What do you notice?
See margin.

The **inverse** of a function is a relation in which the domain and range of the original function are switched. The domain of the original function becomes the range of the inverse and the range of the original function becomes the domain of the inverse. The scatterplot of speed versus braking distance is the inverse of the scatterplot of braking distance versus speed.

7. How are the inverse scatterplot and the original scatterplot related?
The inverse scatterplot is a reflection of the original scatterplot across the line y = x.

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8. A table of values for both scatterplots is shown.

Braking Distance vs. Speed

SPEED, x (MILES PER HOUR)	BRAKING DISTANCE, y (FEET)
20	63
30	109
40	164
50	229
60	303
70	387

Speed vs. Braking Distance

BRAKING DISTANCE, x (FEET)	SPEED, y (MILES PER HOUR)
63	20
109	30
164	40
229	50
303	60
387	70

How are the domain and range of the original scatterplot (braking distance versus speed) and the inverse scatterplot (speed versus braking distance) related?
See margin.

9. Graph the function $f(x) = 3x - 4$. Use a graph and a table of values to represent the inverse of $f(x)$. How do the slope and intercepts of the function compare with the slope and intercepts of the inverse?
See margin.
10. Graph the function $g(x) = (x + 1)^2 + 2$. Use a graph and a table of values to represent the inverse of $g(x)$. How does the vertex of $g(x)$ compare to the vertex of its inverse?
See margin.

REFLECT

- How can you use a graph to generate the inverse of a function?
Reflect the graph of the original function across the line y = x.
- How can you use a table to generate the inverse of a function?
Reverse the independent and dependent variables.
- How are the domain and range of a function related to the domain and range of its inverse?
The domain of the function is the range of its inverse.
The range of the function is the domain of its inverse.
- Is the inverse of a function always a new function? Explain your answer.
See margin.

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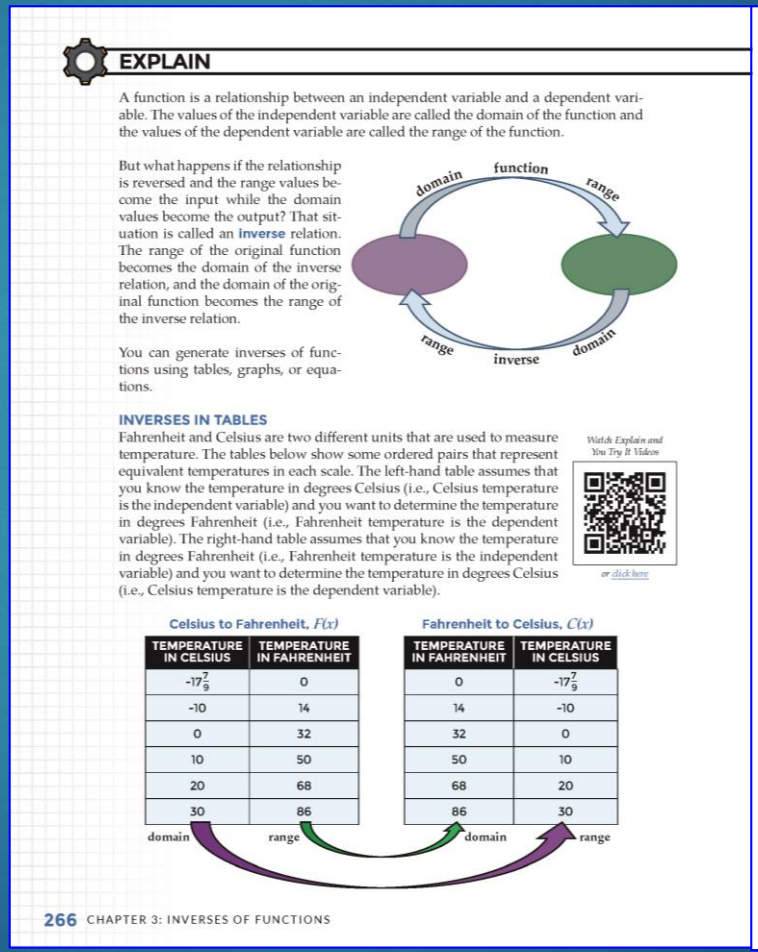
Inquiry-Based Instruction: 5E

- Explain: Use multiple representations to formalize key ideas.
- In this case, inverses are formalized as a reversal of domain and range using graphs, tables, and equations.



EXPLAIN

A function is a relationship between an independent variable and a dependent variable. The values of the independent variable are called the domain of the function and the values of the dependent variable are called the range of the function.

But what happens if the relationship is reversed and the range values become the input while the domain values become the output? That situation is called an **inverse** relation. The range of the original function becomes the domain of the inverse relation, and the domain of the original function becomes the range of the inverse relation.

You can generate inverses of functions using tables, graphs, or equations.

INVERSES IN TABLES

Fahrenheit and Celsius are two different units that are used to measure temperature. The tables below show some ordered pairs that represent equivalent temperatures in each scale. The left-hand table assumes that you know the temperature in degrees Celsius (i.e., Celsius temperature is the independent variable) and you want to determine the temperature in degrees Fahrenheit (i.e., Fahrenheit temperature is the dependent variable). The right-hand table assumes that you know the temperature in degrees Fahrenheit (i.e., Fahrenheit temperature is the independent variable) and you want to determine the temperature in degrees Celsius (i.e., Celsius temperature is the dependent variable).

Watch Explain and You Try It Videos

or click here

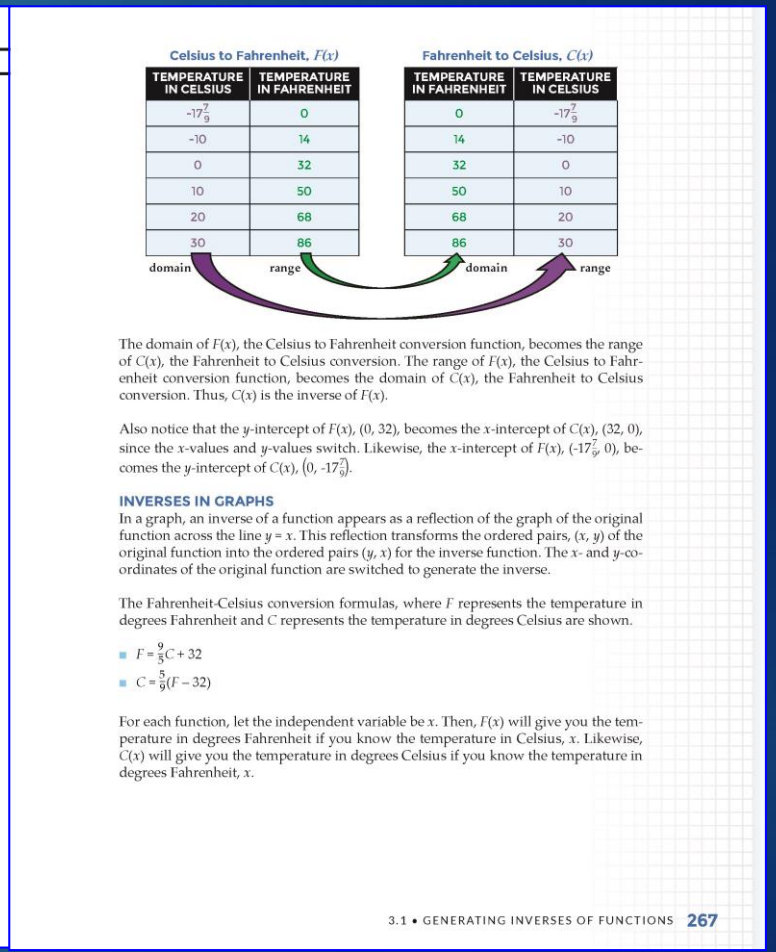
Celsius to Fahrenheit, $F(x)$

TEMPERATURE IN CELSIUS	TEMPERATURE IN FAHRENHEIT
$-17\frac{7}{9}$	0
-10	14
0	32
10	50
20	68
30	86

Fahrenheit to Celsius, $C(x)$

TEMPERATURE IN FAHRENHEIT	TEMPERATURE IN CELSIUS
0	$-17\frac{7}{9}$
14	-10
32	0
50	10
68	20
86	30

domain range domain range



Celsius to Fahrenheit, $F(x)$

TEMPERATURE IN CELSIUS	TEMPERATURE IN FAHRENHEIT
$-17\frac{7}{9}$	0
-10	14
0	32
10	50
20	68
30	86

Fahrenheit to Celsius, $C(x)$

TEMPERATURE IN FAHRENHEIT	TEMPERATURE IN CELSIUS
0	$-17\frac{7}{9}$
14	-10
32	0
50	10
68	20
86	30

domain range domain range

The domain of $F(x)$, the Celsius to Fahrenheit conversion function, becomes the range of $C(x)$, the Fahrenheit to Celsius conversion. The range of $F(x)$, the Celsius to Fahrenheit conversion function, becomes the domain of $C(x)$, the Fahrenheit to Celsius conversion. Thus, $C(x)$ is the inverse of $F(x)$.

Also notice that the y-intercept of $F(x)$, $(0, 32)$, becomes the x-intercept of $C(x)$, $(32, 0)$, since the x-values and y-values switch. Likewise, the x-intercept of $F(x)$, $(-17\frac{7}{9}, 0)$, becomes the y-intercept of $C(x)$, $\left(0, -17\frac{7}{9}\right)$.

INVERSES IN GRAPHS

In a graph, an inverse of a function appears as a reflection of the graph of the original function across the line $y = x$. This reflection transforms the ordered pairs, (x, y) of the original function into the ordered pairs (y, x) for the inverse function. The x- and y-coordinates of the original function are switched to generate the inverse.

The Fahrenheit-Celsius conversion formulas, where F represents the temperature in degrees Fahrenheit and C represents the temperature in degrees Celsius are shown.

- $F = \frac{9}{5}C + 32$
- $C = \frac{5}{9}(F - 32)$

For each function, let the independent variable be x. Then, $F(x)$ will give you the temperature in degrees Fahrenheit if you know the temperature in Celsius, x. Likewise, $C(x)$ will give you the temperature in degrees Celsius if you know the temperature in degrees Fahrenheit, x.

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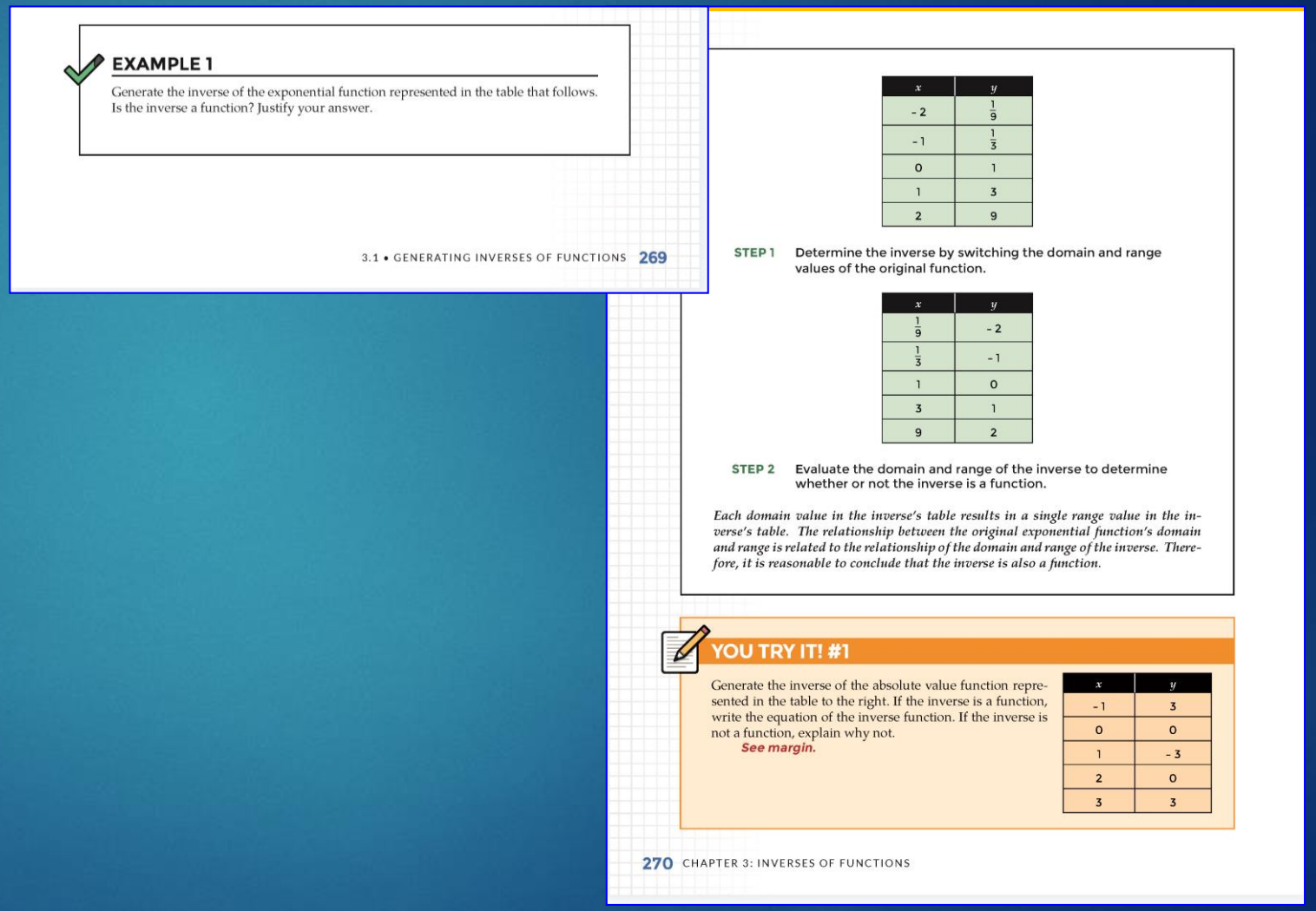
Inquiry-Based Instruction: 5E

- Elaborate: Application or extension of the concept/ideas that were investigated (Explore) and formalized (Explain)
- In this case, the Elaborate is a set of additional examples extending the content and guided-practice style problems.



EXAMPLE 1

Generate the inverse of the exponential function represented in the table that follows. Is the inverse a function? Justify your answer.

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x	y
-2	$\frac{1}{9}$
-1	$\frac{1}{3}$
0	1
1	3
2	9

STEP 1 Determine the inverse by switching the domain and range values of the original function.

x	y
$\frac{1}{9}$	-2
$\frac{1}{3}$	-1
1	0
3	1
9	2

STEP 2 Evaluate the domain and range of the inverse to determine whether or not the inverse is a function.

Each domain value in the inverse's table results in a single range value in the inverse's table. The relationship between the original exponential function's domain and range is related to the relationship of the domain and range of the inverse. Therefore, it is reasonable to conclude that the inverse is also a function.

YOU TRY IT! #1

Generate the inverse of the absolute value function represented in the table to the right. If the inverse is a function, write the equation of the inverse function. If the inverse is not a function, explain why not.

See margin.

x	y
-1	3
0	0
1	-3
2	0
3	3

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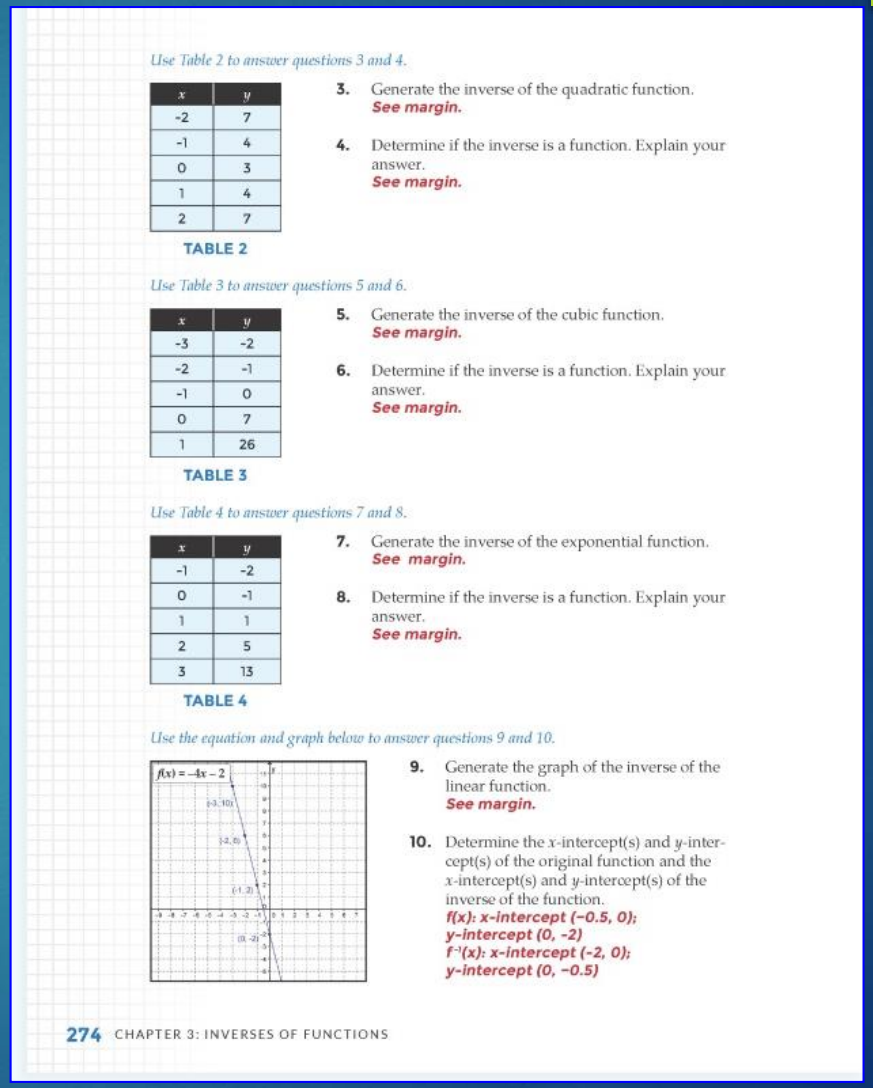
Inquiry-Based Instruction: 5E

- Evaluate: An activity or exercise in which student understanding is determined
- In this case, the Evaluate is a set of independent practice problems.



Use Table 2 to answer questions 3 and 4.

x	y
-2	7
-1	4
0	3
1	4
2	7

TABLE 2

3. Generate the inverse of the quadratic function.
See margin.

4. Determine if the inverse is a function. Explain your answer.
See margin.

Use Table 3 to answer questions 5 and 6.

x	y
-3	-2
-2	-1
-1	0
0	7
1	26

TABLE 3

5. Generate the inverse of the cubic function.
See margin.

6. Determine if the inverse is a function. Explain your answer.
See margin.

Use Table 4 to answer questions 7 and 8.

x	y
-1	-2
0	-1
1	1
2	5
3	13

TABLE 4

7. Generate the inverse of the exponential function.
See margin.

8. Determine if the inverse is a function. Explain your answer.
See margin.

Use the equation and graph below to answer questions 9 and 10.

9. Generate the graph of the inverse of the linear function.
See margin.

10. Determine the x-intercept(s) and y-intercept(s) of the original function and the x-intercept(s) and y-intercept(s) of the inverse of the function.
f(x): x-intercept (−0.5, 0); y-intercept (0, −2)
$f^{-1}(x)$: x-intercept (−2, 0); y-intercept (0, −0.5)

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Summary

- Algebraic Reasoning is a new high school math course created by the Texas State Board of Education.
- Algebraic Reasoning is a rigorous, meaningful course for students who may not be successful in an Algebra 2 course right out of Algebra 1.
- Algebraic Reasoning could be used to support students prior to taking Algebra 2.
- Algebraic Reasoning qualifies as "advanced math" for either the Foundation diploma or an Endorsement.

Thank You!

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