# 3.1 Generating Inverses of Functions

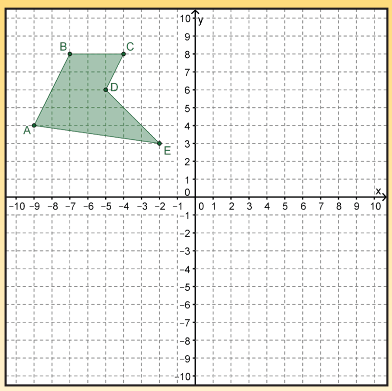
**FOCUSING QUESTION** What is the inverse of a function?

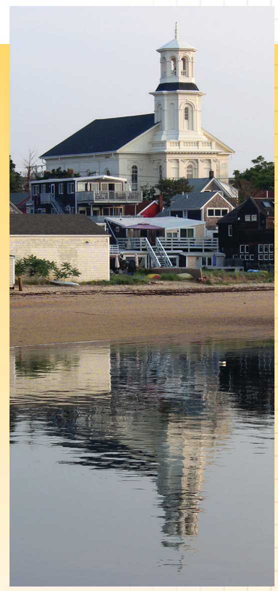
## LEARNING OUTCOMES

- I can compare and contrast the key attributes of a function and its inverse when I have the function as a table, graph, or written symbolically.
- I can represent the domain and range of a linear function in a variety of ways, including interval notation, inequalities, and set builder notation.
- I can use and select tools, including graphing technology, paper and pencil, and manipulatives like patty paper, to solve problems.

## ENGAGE

Dylan, a computer animator, needs to reflect the figure shown as a part of an animation process.



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What would be the coordinates of each vertex in the new figure if Dylan reflected the original figure across the $y$-axis? The $x$-axis?

***Across the y-axis: A'(9, 4) B'(7, 8) C'(4, 8) D'(5, 6) E'(2, 3)***
***Across the x-axis: A'(-9, -4) B'(-7, -8) C'(-4, -8) D'(-5, -6) E'(-2, -3)***

## EXPLORE

The distance required to stop a moving vehicle is a function of the speed of the vehicle. According to the Texas Driver Handbook, the distance required to stop a vehicle moving at a given speed, on dry pavement with good tires, is shown in the table on page 264.



### INTEGRATE TECHNOLOGY

Use the list editor of a graphing calculator to enter the $x$-coordinates and $y$-coordinates of the figure into two separate lists. Use list operations to show how changing the sign of one set of coordinates reflects the figure across either the $x$-axis or $y$-axis.

### TEKS

**AR.3B** Compare and contrast the key attributes of a function and its inverse when it exists, including domain, range, maxima, minima, and intercepts, tabularly, graphically, and symbolically.

**AR.7A** Represent domain and range of a function using interval notation, inequalities, and set (builder) notation.

### MATHEMATICAL PROCESS SPOTLIGHT

**AR.1C** Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems.

### ELPS

**4F** Use visual and contextual support and support from peers and teachers to read grade-appropriate content area text, enhance and confirm understanding, and develop vocabulary, grasp of language structures, and background knowledge needed to comprehend increasingly challenging language

### VOCABULARY

function, inverse, domain, range, maximum, minimum, $x$-intercept, $y$-intercept, line of reflection

### MATERIALS

- graphing calculator
- graph paper
- patty paper

| SPEED (MILES PER HOUR) | 20 | 30 | 40 | 50 | 60 | 70 |
|---|---|---|---|---|---|---|
| BRAKING DISTANCE (FEET) | 63 | 109 | 164 | 229 | 303 | 387 |

Source: Texas Department of Public Safety

**1.** Use finite differences or successive ratios to determine if the data set represents a linear, quadratic, cubic, or exponential function.
***The second differences are approximately constant (9.5) so the data set represents a quadratic function.***

**2.** Make a scatterplot of the braking distance versus speed.
***See margin.***

**3.** Suppose that you knew from a skid mark the length of the braking distance and wanted to know the speed of the moving vehicle. In this situation, what would be the independent variable and what would be the dependent variable?
***Braking distance is the independent variable and speed is the dependent variable.***

> When describing a scatterplot, you can say the scatterplot is the name of the **dependent variable** versus the name of the **independent variable**. For example, if you are graphing data where **speed is the independent variable** and **distance is the dependent variable**, you can say that you have a scatterplot of ***distance*** versus ***speed***.

**4.** On the same grid, make a scatterplot of speed versus braking distance. What do you notice about the two scatterplots?
***See margin.***

**5.** Draw the line $y = x$ on your graph. Lay a sheet of patty paper on top of the graph. Trace and label the $x$-axis, $y$-axis, the line $y = x$, and the points in the first scatter plot, braking distance versus speed.
***See margin.***

**6.** Use the patty paper to reflect the first scatterplot across the line $y = x$. *(Hint: Hold the patty paper along the top-right corner of the line $y = x$ with your right index finger and thumb. Hold the patty paper along the bottom-left corner of the line $y = x$ with your left index finger and thumb. Flip the patty paper over without moving your fingers and thumbs. Line up the axes on the patty paper with the axes on the graph beneath.)* What do you notice?
***See margin.***

The **inverse** of a function is a relation in which the domain and range of the original function are switched. The domain of the original function becomes the range of the inverse and the range of the original function becomes the domain of the inverse. The scatterplot of speed versus braking distance is the inverse of the scatterplot of braking distance versus speed.

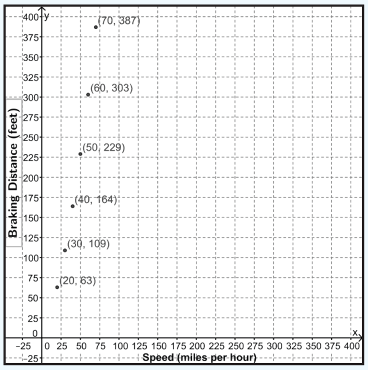
**7.** How are the inverse scatterplot and the original scatterplot related?
***The inverse scatterplot is a reflection of the original scatterplot across the line y = x.***



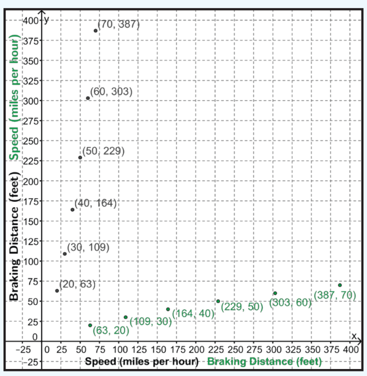
## INTEGRATE TECHNOLOGY

Use the list editor of a graphing calculator to enter the data into two separate lists (List 1 for the independent variable and List 2 for the dependent variable). Make a scatterplot of List 2 versus List 1, then reverse the domain and range to make a scatterplot of List 1 versus List 2. Compare the two scatterplots visually. Graph the line $y = x$ on top of the scatterplots and determine whether or not the two scatterplots are a reflection of each other across the line $y = x$.

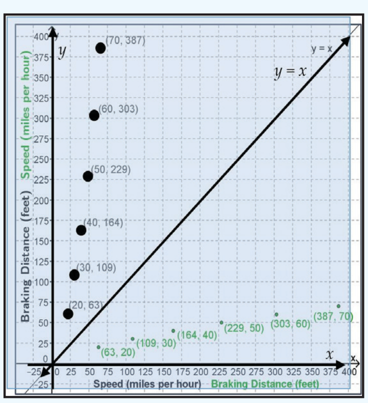
**2.**



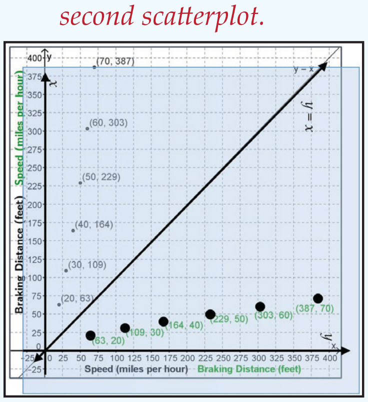

**4.** *The second scatterplot, speed versus braking distance, appears to be a reflection across a diagonal line of the first scatterplot, braking distance versus speed.*




**5.**




**6.** *The points from the first scatterplot line up exactly on top of the points on the second scatterplot.*

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**8.** A table of values for both scatterplots is shown.

**Braking Distance vs. Speed**

| SPEED, $x$ (MILES PER HOUR) | BRAKING DISTANCE, $y$ (FEET) |
|---|---|
| 20 | 63 |
| 30 | 109 |
| 40 | 164 |
| 50 | 229 |
| 60 | 303 |
| 70 | 387 |

**Speed vs. Braking Distance**

| BRAKING DISTANCE, $x$ (FEET) | SPEED, $y$ (MILES PER HOUR) |
|---|---|
| 63 | 20 |
| 109 | 30 |
| 164 | 40 |
| 229 | 50 |
| 303 | 60 |
| 387 | 70 |

How are the domain and range of the original scatterplot (braking distance versus speed) and the inverse scatterplot (speed versus braking distance) related?
***See margin.***

**9.** Graph the function $f(x) = 3x - 4$. Use a graph and a table of values to represent the inverse of $f(x)$. How do the slope and intercepts of the function compare with the slope and intercepts of the inverse?
***See margin.***

**10.** Graph the function $g(x) = (x + 4)^2 + 2$. Use a graph and a table of values to represent the inverse of $g(x)$. How does the vertex of $g(x)$ compare to the vertex of its inverse?
***See margin.***

## REFLECT

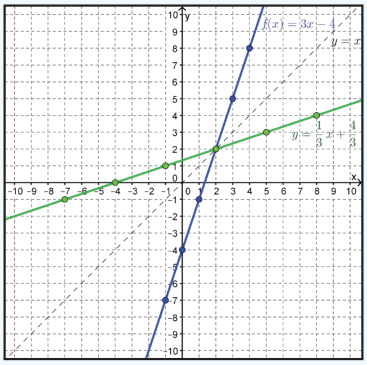
- **How can you use a graph to generate the inverse of a function?**
  ***Reflect the graph of the original function across the line y = x.***
- **How can you use a table to generate the inverse of a function?**
  ***Reverse the independent and dependent variables.***
- **How are the domain and range of a function related to the domain and range of its inverse?**
  ***The domain of the function is the range of its inverse.***
  ***The range of the function is the domain of its inverse.***
- **Is the inverse of a function always a new function? Explain your answer.**
  ***See margin.***



**REFLECT ANSWER:**
*No. A function is a relation in which each value of the independent variable generates only one value of the dependent variable. The inverse of a linear function is always a function. But the inverse of a quadratic function is not a function because, except for the vertex, each independent variable value generates two values of the dependent variable.*

**8.** *The domain of the original scatterplot becomes the range of the inverse scatterplot. The range of the original scatterplot becomes the domain of the inverse scatterplot.*

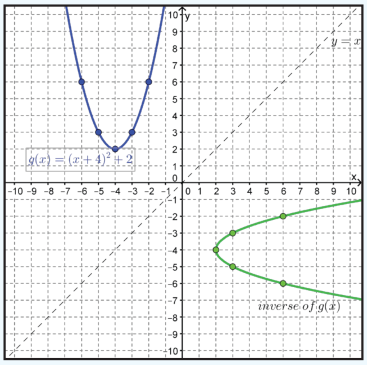
**9.** *The slope of the inverse is the reciprocal of the slope of the function, f(x). The y-coordinate of the y-intercept of f(x) is the same as the x-coordinate of the x-intercept of its inverse. The x-coordinate of the x-intercept of f(x) is the same as the y-coordinate of the y-intercept of its inverse.*




| $x$ | $f(x) = 3x - 4$ |
|---|---|
| -1 | -7 |
| 0 | -4 |
| 1 | -1 |
| 2 | 2 |
| 3 | 5 |
| 4 | 8 |

| $x$ | $y$ |
|---|---|
| -7 | -1 |
| -4 | 0 |
| -1 | 1 |
| 2 | 2 |
| 5 | 3 |
| 8 | 4 |

**10.** *The coordinates of the vertex of g(x) are switched for the vertex of its inverse.*




| $x$ | $g(x) = (x - 4)^2 + 2$ |
|---|---|
| -1 | -7 |
| 0 | -4 |
| 1 | -1 |
| 2 | 2 |
| 3 | 5 |
| 4 | 8 |

| $x$ | $y$ |
|---|---|
| -7 | -1 |
| -4 | 0 |
| -1 | 1 |
| 2 | 2 |
| 5 | 3 |
| 8 | 4 |

## QUESTIONING STRATEGIES

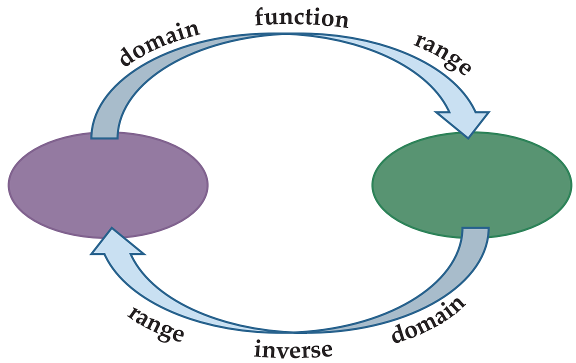
As students begin to find the inverses of functions, it is important to call back to prior learning about functions from previous courses.

- How can I tell if a graph represents a function?
- How can I tell if a table of values represents a function?
- How can I tell if an equation represents a function?

# EXPLAIN

A function is a relationship between an independent variable and a dependent variable. The values of the independent variable are called the domain of the function and the values of the dependent variable are called the range of the function.

But what happens if the relationship is reversed and the range values become the input while the domain values become the output? That situation is called an **inverse** relation. The range of the original function becomes the domain of the inverse relation, and the domain of the original function becomes the range of the inverse relation.




You can generate inverses of functions using tables, graphs, or equations.

## INVERSES IN TABLES

Fahrenheit and Celsius are two different units that are used to measure temperature. The tables below show some ordered pairs that represent equivalent temperatures in each scale. The left-hand table assumes that you know the temperature in degrees Celsius (i.e., Celsius temperature is the independent variable) and you want to determine the temperature in degrees Fahrenheit (i.e., Fahrenheit temperature is the dependent variable). The right-hand table assumes that you know the temperature in degrees Fahrenheit (i.e., Fahrenheit temperature is the independent variable) and you want to determine the temperature in degrees Celsius (i.e., Celsius temperature is the dependent variable).

*Watch Explain and You Try It Videos*

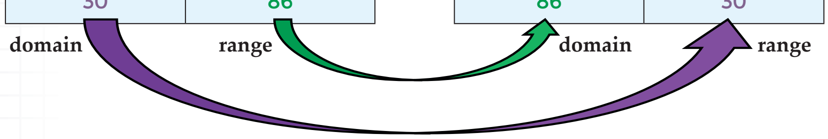
*or click here*

**Celsius to Fahrenheit, $F(x)$**

| TEMPERATURE IN CELSIUS | TEMPERATURE IN FAHRENHEIT |
|---|---|
| $-17\frac{7}{9}$ | 0 |
| -10 | 14 |
| 0 | 32 |
| 10 | 50 |
| 20 | 68 |
| 30 | 86 |

**Fahrenheit to Celsius, $C(x)$**

| TEMPERATURE IN FAHRENHEIT | TEMPERATURE IN CELSIUS |
|---|---|
| 0 | $-17\frac{7}{9}$ |
| 14 | -10 |
| 32 | 0 |
| 50 | 10 |
| 68 | 20 |
| 86 | 30 |



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**Celsius to Fahrenheit, $F(x)$**

| TEMPERATURE IN CELSIUS | TEMPERATURE IN FAHRENHEIT |
|---|---|
| $-17\frac{7}{9}$ | 0 |
| -10 | 14 |
| 0 | 32 |
| 10 | 50 |
| 20 | 68 |
| 30 | 86 |

domain range

**Fahrenheit to Celsius, $C(x)$**

| TEMPERATURE IN FAHRENHEIT | TEMPERATURE IN CELSIUS |
|---|---|
| 0 | $-17\frac{7}{9}$ |
| 14 | -10 |
| 32 | 0 |
| 50 | 10 |
| 68 | 20 |
| 86 | 30 |

domain range

The domain of $F(x)$, the Celsius to Fahrenheit conversion function, becomes the range of $C(x)$, the Fahrenheit to Celsius conversion. The range of $F(x)$, the Celsius to Fahrenheit conversion function, becomes the domain of $C(x)$, the Fahrenheit to Celsius conversion. Thus, $C(x)$ is the inverse of $F(x)$.

Also notice that the $y$-intercept of $F(x)$, (0, 32), becomes the $x$-intercept of $C(x)$, (32, 0), since the $x$-values and $y$-values switch. Likewise, the $x$-intercept of $F(x)$, $(-17\frac{7}{9}, 0)$, becomes the $y$-intercept of $C(x)$, $\left(0, -17\frac{7}{9}\right)$.

## INVERSES IN GRAPHS

In a graph, an inverse of a function appears as a reflection of the graph of the original function across the line $y = x$. This reflection transforms the ordered pairs, $(x, y)$ of the original function into the ordered pairs $(y, x)$ for the inverse function. The $x$- and $y$-coordinates of the original function are switched to generate the inverse.

The Fahrenheit-Celsius conversion formulas, where $F$ represents the temperature in degrees Fahrenheit and $C$ represents the temperature in degrees Celsius are shown.

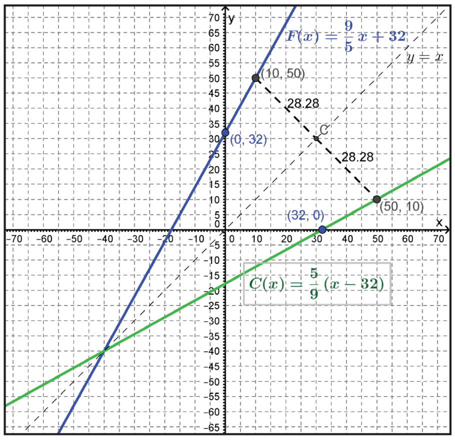
- $F = \frac{9}{5}C + 32$
- $C = \frac{5}{9}(F - 32)$

For each function, let the independent variable be $x$. Then, $F(x)$ will give you the temperature in degrees Fahrenheit if you know the temperature in Celsius, $x$. Likewise, $C(x)$ will give you the temperature in degrees Celsius if you know the temperature in degrees Fahrenheit, $x$.

## EXTENDING STUDENT UNDERSTANDING

A third temperature scale, Kelvin, measures temperature with respect to absolute zero, which is the temperature at which all molecules stop moving. $K = C - 273$, where $K$ represents the temperature in Kelvins and C represents the temperature in degrees Celsius. Ask students to make a table, graph, and equation of this relationship and generate its inverse from each of the three representations.

Notice that each line is a reflection of the other across the line $y = x$. Graphically, reflections have the property that each point on the graph of the function is the same distance from the line of reflection as its image, or reflection, on the graph of the inverse. For example, (10, 50) represents the equivalent temperatures 10°C and 50°F. This point lies 28.28 units from the line $y = x$, which is the line of reflection. Its image, (50, 10), represents the equivalent temperatures 50°F and 10°C. This point also lies 28.28 units from the line $y = x$. Thus, the points are equidistant, or the same distance, from the line of reflection.




Also notice the relationships between the intercepts.

- The $y$-intercept of $F(x)$ becomes the $x$-intercept of $C(x)$. In other words, the point (0, 32) on the graph of the original function becomes the point (32, 0) on the graph of the inverse because the $x$- and $y$-coordinates for the inverse are switched from the original function.
- The $x$-intercept of $F(x)$ becomes the $y$-intercept of $C(x)$. In other words, the point $\left(-17\frac{7}{9}, 0\right)$ on the graph of the original function becomes the point $\left(0, -17\frac{7}{9}\right)$ on the graph of the inverse because the $x$- and $y$-coordinates for the inverse are switched from the original function.

## INVERSES IN EQUATIONS

The Fahrenheit-Celsius conversion formulas, where $F$ represents the temperature in degrees Fahrenheit and $C$ represents the temperature in degrees Celsius are shown.

- $F = \frac{9}{5}C + 32$
- $C = \frac{5}{9}(F - 32)$

You can determine the inverse of a function from its equation. Because the domain and range of the original function are switched to generate the inverse, switch the variables that represent the independent and dependent variables in the equation and solve for the dependent variable.

Begin with the Celsius to Fahrenheit conversion formula written as a function, $F(x) = \frac{9}{5}x + 32$. $x$ represents the independent variable and $F(x)$ represents the dependent variable, which can also be rewritten as $y$. For its inverse, switch $x$ and $y$ and then solve for $y$.

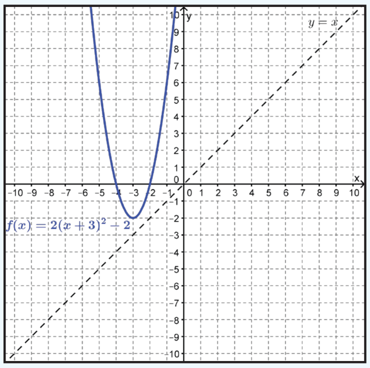


## DIFFERENTIATING INSTRUCTION

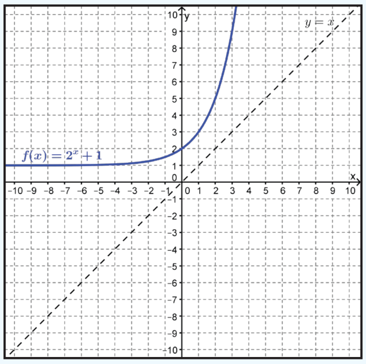
Marian Small and Amy Lin (*More Good Questions: Great Ways to Differentiate Mathematics*) recommend using parallel tasks and open questions as differentiation structures to make mathematical tasks accessible to all students. Parallel tasks are two tasks that accomplish the same learning outcome with different levels of complexity. Students select one of the two tasks based on their comfort level with the topic. In this lesson, parallel tasks that could be posed to help students make sense of the Explain content include:

Sketch the graph of the inverse of the function with the graph shown.

**OPTION 1:**




**OPTION 2:**

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$$y = \frac{9}{5}x + 32$$

$$x = \frac{9}{5}y + 32$$

$$x - 32 = \frac{9}{5}y + 32 - 32$$

$$x - 32 = \frac{9}{5}y$$

$$\frac{5}{9}(x - 32) = \frac{5}{9}\left(\frac{9}{5}\right)y$$

$$\frac{5}{9}(x - 32) = y$$

In this inverse equation, $y$ represents $C(x)$, which is the temperature in degrees Celsius, and $x$ represents the temperature in degrees Fahrenheit. This equation, which is a linear function, is equivalent to the Fahrenheit to Celsius temperature conversion formula.

**KEY ATTRIBUTES OF INVERSES OF FUNCTIONS**

The inverse of a function may or may not be a function itself. However, it does have several important key attributes:

- The domain of the original function is the range of the inverse.
- The range of the original function is the domain of the inverse.
- If the inverse of $f(x)$ is a function, it can be written with the notation $f^{-1}(x)$, which is read, "f inverse of x."
- The $x$-intercept(s) of the original function is/are the $y$-intercept(s) of the inverse.
- The $y$-intercept of the original function is the $x$-intercept of the inverse.
- The maximum or minimum function value ($y$-value) of the original function (e.g., vertex of a quadratic function) is the maximum or minimum $x$-value of the inverse.

## EXAMPLE 1

Generate the inverse of the exponential function represented in the table that follows. Is the inverse a function? Justify your answer.



## ADDITIONAL EXAMPLES

For each of the tables of values below, do the values represent a function? Justify your answer.

**1.**

| $x$ | $y$ |
|---|---|
| 0 | -1 |
| 4 | 3 |
| 6 | 5 |
| 8 | 5 |
| 12 | 1 |

*Function*

**2.**

| $x$ | $y$ |
|---|---|
| 4 | 0 |
| 5 | 5 |
| 6 | 10 |
| 5 | 15 |
| 4 | 10 |

*Not a function*

**3.**

| $x$ | $y$ |
|---|---|
| -10 | 7 |
| -4 | 7 |
| 2 | 7 |
| 8 | 7 |
| 14 | 7 |

*Function*

**4.**

| $x$ | $y$ |
|---|---|
| -2 | -10 |
| -1 | -5 |
| 0 | 0 |
| 1 | -4 |
| 2 | -10 |

*Function*

## ADDITIONAL EXAMPLES

**1.** Generate the inverse of the quadratic function represented in the table below. Is the inverse a function? Justify your answer.

| *x* | *y* |
|---|---|
| -2 | -156 |
| -1 | -44 |
| 0 | 4 |
| 1 | -12 |
| 2 | -92 |

| *x* | *y* |
|---|---|
| ***-156*** | ***-2*** |
| ***-44*** | ***-1*** |
| ***4*** | ***0*** |
| ***-12*** | ***1*** |
| ***-92*** | ***2*** |

*Not a function*

**2.** Generate the inverse of the cubic function represented in the table below. Is the inverse a function? Justify your answer.

| *x* | *y* |
|---|---|
| 4 | -9.5 |
| 8 | 0 |
| 12 | 3.5 |
| 16 | 4 |
| 20 | 4.5 |
| 24 | 8 |

| *x* | *y* |
|---|---|
| -9.5 | 4 |
| 0 | 8 |
| 3.5 | 12 |
| 4 | 16 |
| 4.5 | 20 |
| 8 | 24 |

*Function*

| *x* | *y* |
|---|---|
| – 2 | $\frac{1}{9}$ |
| – 1 | $\frac{1}{3}$ |
| 0 | 1 |
| 1 | 3 |
| 2 | 9 |

**STEP 1** Determine the inverse by switching the domain and range values of the original function.

| *x* | *y* |
|---|---|
| $\frac{1}{9}$ | – 2 |
| $\frac{1}{3}$ | – 1 |
| 1 | 0 |
| 3 | 1 |
| 9 | 2 |

**STEP 2** Evaluate the domain and range of the inverse to determine whether or not the inverse is a function.

*Each domain value in the inverse's table results in a single range value in the inverse's table. The relationship between the original exponential function's domain and range is related to the relationship of the domain and range of the inverse. Therefore, it is reasonable to conclude that the inverse is also a function.*

### YOU TRY IT! #1

Generate the inverse of the absolute value function represented in the table to the right. If the inverse is a function, write the equation of the inverse function. If the inverse is not a function, explain why not.

***See margin.***

| *x* | *y* |
|---|---|
| – 1 | 3 |
| 0 | 0 |
| 1 | – 3 |
| 2 | 0 |
| 3 | 3 |



**YOU TRY IT! #1 ANSWER:**

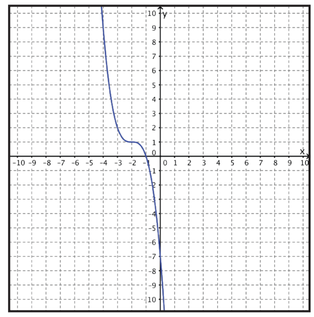
*The inverse of the absolute value function is represented in the table:*

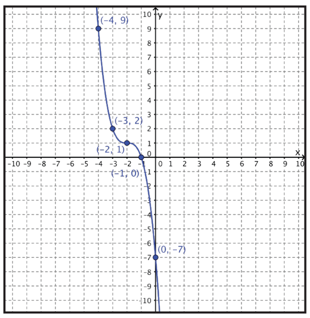
| *x* | *y* |
|---|---|
| ***3*** | ***-1*** |
| ***0*** | ***0*** |
| ***-3*** | ***1*** |
| ***0*** | ***2*** |
| ***3*** | ***3*** |

*The inverse is not a function because more than one range value results from the same domain value. The domain value of 0 is associated with the range values 0 and 2 and the domain value of 3 is associated with the range values – 1 and 3.*

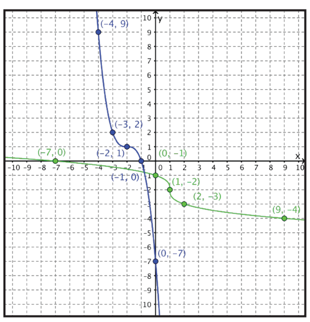
## EXAMPLE 2

Generate the graph of the inverse of the cubic function $f(x) = -(x + 2)^3 + 1$ whose graph is shown below.

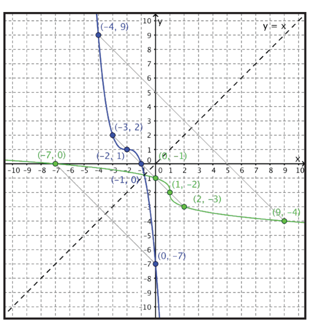




**STEP 1** Determine several points on the original graph that represent the general shape of the curve as well as key attributes of the function, such as intercepts.



**STEP 2** Switch the $x$- and $y$-values of each of the chosen points from the original graph and graph them on the coordinate plane. Then draw the curve of the inverse through the points you graphed on the coordinate plane.



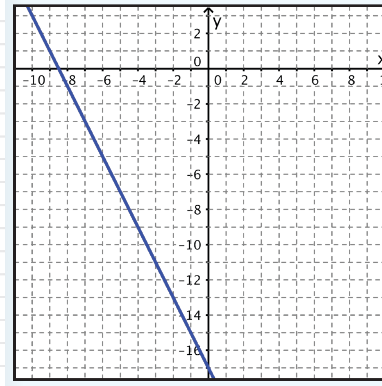
**STEP 3** Verify visually that the graphs are reflections of one another across the line $y = x$ and that the graphed points are the same distance from the line $y = x$.

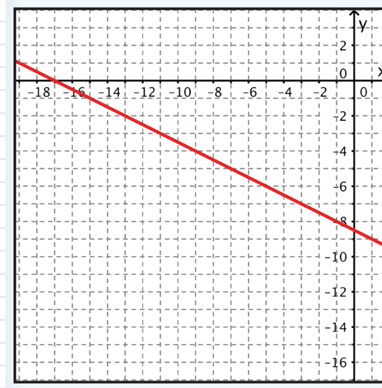
Notice that the $y$-intercept of the original function is the $x$-intercept of the inverse and the $x$-intercept(s) of the original function are the $y$-intercept(s) of the inverse.



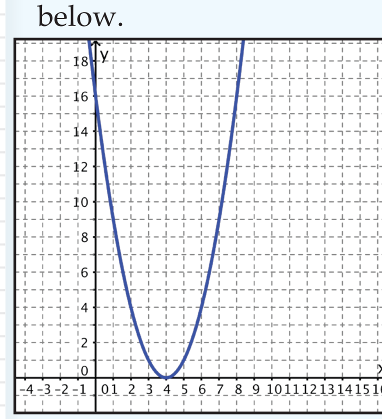
### ADDITIONAL EXAMPLES

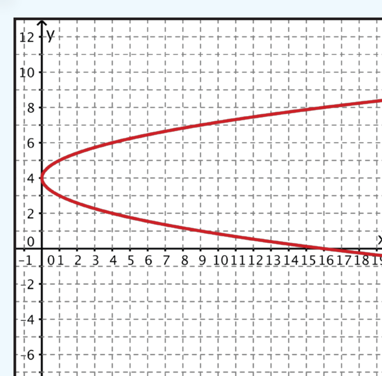
**1.** Generate the graph of the inverse of the linear function $g(x) = -2(x + 6) - 5$ whose graph is shown below.



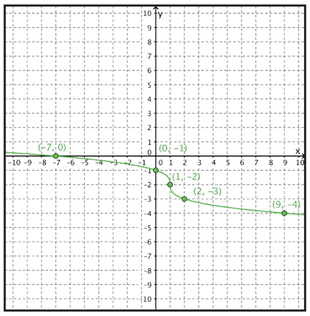


**2.** Generate the graph of the inverse of the quadratic function $h(x) = \frac{1}{4}(2x - 8)^2$ whose graph is shown below.





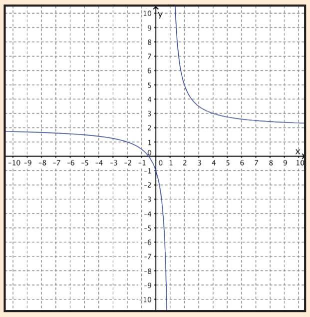
The graph of the inverse of $f^{-1}(x)$ is shown.



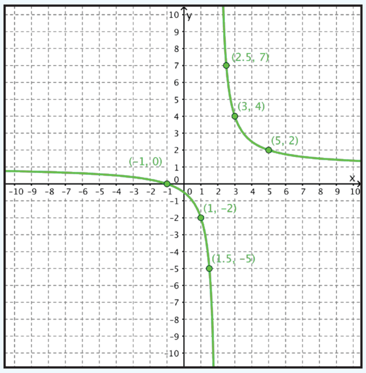

## YOU TRY IT! #2

Generate the graph of the inverse of the rational function $g(x) = \frac{3}{x-1} + 2$ whose graph is shown below.

***See margin.***



**YOU TRY IT! #2 ANSWER:**




## EXAMPLE 3

Generate the equation of $h^{-1}(x)$, the inverse of the quadratic function $h(x) = 3(2x + 1)^2 - 7$.

**STEP 1** Rewrite the function using $x$ as the independent variable and $y$ as the dependent variable.

$$y = 3(2x + 1)^2 - 7$$

**STEP 2** Reverse the variables $x$ and $y$ in the equation and solve the equation for $y$.



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$$x = 3(2y+1)^2 - 7$$

$$x + 7 = 3(2y+1)^2 - 7 + 7$$

$$\frac{x+7}{3} = \frac{3(2y+1)^2}{3}$$

$$\pm\sqrt{\frac{x+7}{3}} = \sqrt{(2y+1)^2}$$

$$-1 \pm \sqrt{\frac{x+7}{3}} = 2y + 1 - 1$$

$$\frac{-1 \pm \sqrt{\frac{x+7}{3}}}{2} = \frac{2y}{2}$$

$$\frac{-1 \pm \sqrt{\frac{x+7}{3}}}{2} = y$$

**STEP 3** **Rewrite the inverse as an equation.**

inverse of $h(x)$ is $y = \frac{-1 \pm \sqrt{\frac{x+7}{3}}}{2}$

## YOU TRY IT! #3

Generate the equation of $p^{-1}(x)$, the inverse of the linear function $p(x) = \frac{2}{5}(x - 3) + 4$.

***$p^{-1}(x) = \frac{5}{2}(x - 4) + 3$***

## PRACTICE/HOMEWORK

*Use Table 1 to answer questions 1 and 2.*

| $x$ | $y$ |
|---|---|
| -1 | -2 |
| 0 | 1 |
| 1 | 4 |
| 2 | 7 |
| 3 | 10 |

**TABLE 1**

**1.** Generate the inverse of the linear function.
***See margin.***

**2.** Determine if the inverse is a function. Explain your answer.
***See margin.***



### ADDITIONAL EXAMPLES

**1.** Generate the equation of $p^{-1}(x)$, the inverse of the linear function $p(x) = -6(\frac{1}{2}x + 5) - 12$.

*$p^{-1}(x) = 2\left(\frac{x+12}{-6} - 5\right)$, or*

*$p^{-1}(x) = 2\left(\frac{x+12}{-6}\right) - 10$*

**2.** Generate the equation of $q^{-1}(x)$, the inverse of the quadratic function $q(x) = \frac{1}{2}(x - 4)^2 + 10$.

*$q^{-1}(x) = 4 + \sqrt{2(x-10)}$*

**3.** Generate the equation of $r^{-1}(x)$, the inverse of the cubic function $r(x) = (-3x - 9)^3 - 5$.

*$r^{-1}(x) = -3 - \frac{\sqrt[3]{x+5}}{3}$, or*

*$r^{-1}(x) = \frac{9 + \sqrt[3]{x+5}}{-3}$*

**1.**

| $x$ | $y$ |
|---|---|
| -2 | -1 |
| 1 | 0 |
| 4 | 1 |
| 7 | 2 |
| 10 | 3 |

**2.** *The inverse is a function because each x-value generates only one y-value.*

**3.**

| *x* | *y* |
|---|---|
| 7 | -2 |
| 4 | -1 |
| 3 | 0 |
| 4 | 1 |
| 7 | 2 |

**4.** *The inverse is not a function because when $x = 4$, $y =$ both $-1$ and $1$.*

**5.**

| *x* | *y* |
|---|---|
| -2 | -3 |
| -1 | -2 |
| 0 | -1 |
| 7 | 0 |
| 26 | 1 |

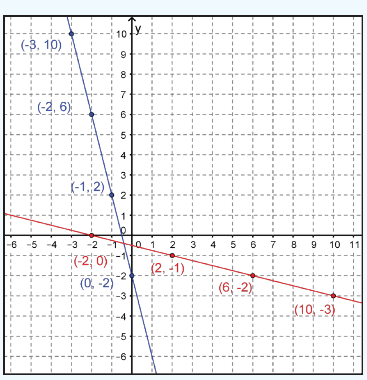
**6.** *The inverse is a function because each x-value generates only one y-value.*

**7.**

| *x* | *y* |
|---|---|
| -2 | -1 |
| -1 | 0 |
| 1 | 1 |
| 5 | 2 |
| 13 | 3 |

**8.** *The inverse is a function because each x-value generates only one y-value.*

**9.**




*Use Table 2 to answer questions 3 and 4.*

| *x* | *y* |
|---|---|
| -2 | 7 |
| -1 | 4 |
| 0 | 3 |
| 1 | 4 |
| 2 | 7 |

**TABLE 2**

**3.** Generate the inverse of the quadratic function.
***See margin.***

**4.** Determine if the inverse is a function. Explain your answer.
***See margin.***

*Use Table 3 to answer questions 5 and 6.*

| *x* | *y* |
|---|---|
| -3 | -2 |
| -2 | -1 |
| -1 | 0 |
| 0 | 7 |
| 1 | 26 |

**TABLE 3**

**5.** Generate the inverse of the cubic function.
***See margin.***

**6.** Determine if the inverse is a function. Explain your answer.
***See margin.***

*Use Table 4 to answer questions 7 and 8.*

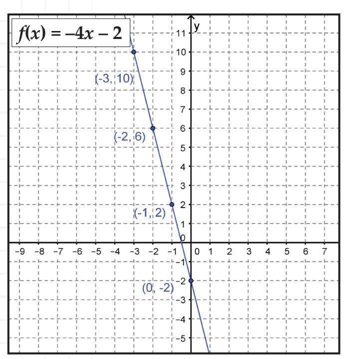
| *x* | *y* |
|---|---|
| -1 | -2 |
| 0 | -1 |
| 1 | 1 |
| 2 | 5 |
| 3 | 13 |

**TABLE 4**

**7.** Generate the inverse of the exponential function.
***See margin.***

**8.** Determine if the inverse is a function. Explain your answer.
***See margin.***

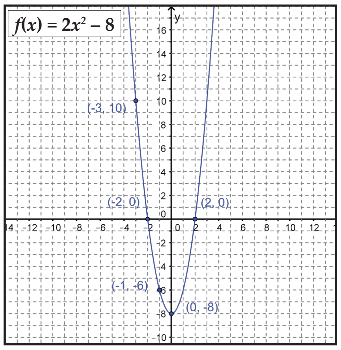
*Use the equation and graph below to answer questions 9 and 10.*




**9.** Generate the graph of the inverse of the linear function.
***See margin.***

**10.** Determine the *x*-intercept(s) and *y*-intercept(s) of the original function and the *x*-intercept(s) and *y*-intercept(s) of the inverse of the function.
***$f(x)$: x-intercept (−0.5, 0); y-intercept (0, -2)***
***$f^{-1}(x)$: x-intercept (-2, 0); y-intercept (0, −0.5)***

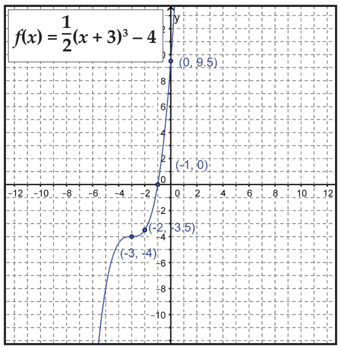
*Use the equation and graph below to answer questions 11 and 12.*




**11.** Generate the graph of the inverse of the quadratic function.
***See margin.***

**12.** Determine the $x$-intercept(s) and $y$-intercept(s) of the original function and the $x$-intercept(s) and $y$-intercept(s) of the inverse of the function.
***f(x): x-intercepts (-2, 0) and (2, 0);***
***y-intercept (0, -8)***
***$f^{-1}(x)$: x-intercept (-8, 0);***
***y-intercepts (0, -2) and (0, 2)***

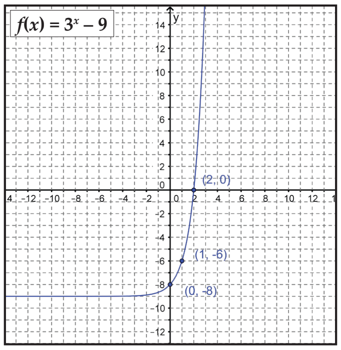
*Use the equation and graph below to answer questions 13 and 14.*




**13.** Generate the graph of the inverse of the cubic function.
***See margin.***

**14.** Determine the $x$-intercept(s) and $y$-intercept(s) of the original function and the $x$-intercept(s) and $y$-intercept(s) of the inverse of the function.
***f(x): x-intercept (-1, 0);***
***y-intercept (0, 9.5)***
***$f^{-1}(x)$: x-intercept (9.5, 0);***
***y-intercept (0, -1)***

*Use the equation and graph below to answer questions 15 and 16.*




**15.** Generate the graph of the inverse of the exponential function.
***See margin.***

**16.** Determine the $x$-intercept(s) and $y$-intercept(s) of the original function and the x-intercept(s) and $y$-intercept(s) of the inverse of the function.
***f(x): x-intercept (2, 0);***
***y-intercept (0, -8)***
***$f^{-1}(x)$: x-intercept (-8, 0);***
***y-intercept (0, 2)***

*For questions 17 – 20, generate $f^{-1}(x)$, the equation of the inverse of the function given.*

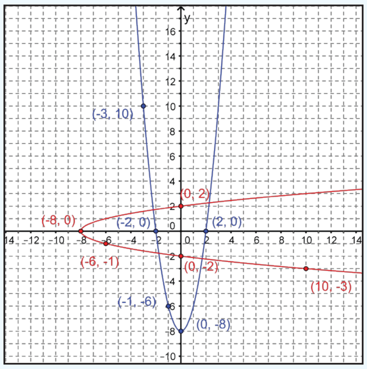
**17.** $f(x) = 3x - 2$
***$f^{-1}(x) = \frac{x + 2}{3}$***

**18.** $f(x) = \frac{1}{4}(8x + 2)$
***$f^{-1}(x) = \frac{2x - 1}{4}$***

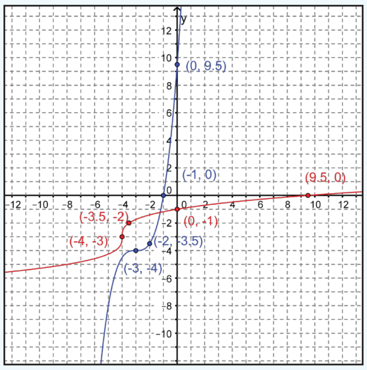
**19.** $f(x) = -2(3x + 1)^2$
***$f^{-1}(x) = y = \frac{-1 \pm \sqrt{\frac{x}{-2}}}{3}$***

**20.** $f(x) = \frac{1}{2}(2x + 5)^2$
***$f^{-1}(x) = y = \frac{-5 \pm \sqrt{2x}}{2}$***

**11.**




**13.**




**15.**