

Powerful Ideas with
Multiplication and Division

Grades 6-8

RGVCTM

The Rio Grande Valley Council of Teachers of Mathematics



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Council of Teachers of Mathematics

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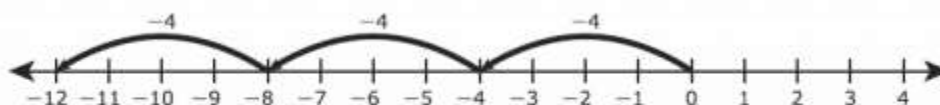


6.3(C) The student is expected to represent integer operations with concrete models and connect the actions with the models to standardized algorithms.

All questions shown below are the sole property of the Texas Education Agency (TEA) and are listed here for informational purposes only.

Question 10- 2015

10 Which expression is represented on the number line?

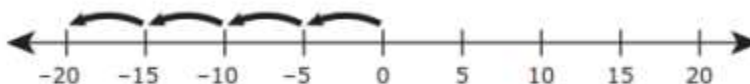


- A $-12 \div -3$
- B $0 - (-12)$
- C $-3 \cdot -4$
- D $3(-4)$

Answer : D

Question 12- 2014

12 Which expression is represented by the model below?



- F $-4 \cdot (-5)$
- G $4 \cdot (-5)$
- H $(-4) \cdot 5$
- J $4 \cdot 5$

Answer : G



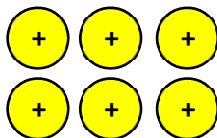
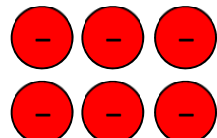
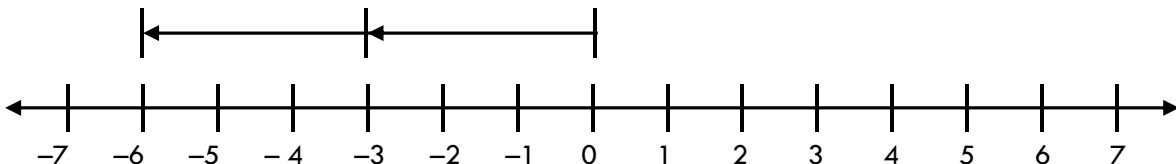
Multiplying and Dividing Integers

Explore

Directions: Use two color counters to to build each array shown. For each negative, flip the counters, and then determine the value of the resulting array. Complete the table by filling in the representations of the problem.



1.

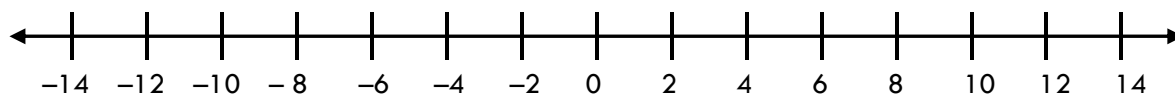
	2×-3	Value	Number Sentence	Verbal Description
Start		-6	$2 \times -3 = -6$	I started with 2 rows of 3 counters. The negative indicated to take the opposite, so I flipped the 6 counters.
Finish				
Solution Using a Number Line				
				



2.

	-3×4	Value	Number Sentence	Verbal Description
Start				
Finish				

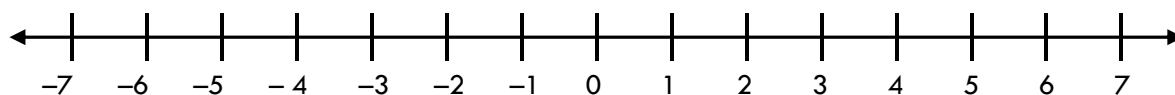
Solution Using a Number Line



3.

	-2×-3	Value	Number Sentence	Verbal Description
Start				
Finish				

Solution Using a Number Line



Use patterns to complete the tables below.

Multiplication			Related Division	
3×5	15	so	$15 \div 5$	3
2×5	10	so	$10 \div 5$	2
1×5	5	so	$5 \div 5$	1
0×5		so	$0 \div 5$	
-1×5		so	$-5 \div 5$	
-2×5		so	$-10 \div 5$	
-3×5		so	$-15 \div 5$	

Multiplication			Related Division	
$3 \times (-5)$	-15	so	$-15 \div (-5)$	3
$2 \times (-5)$	-10	so	$-10 \div (-5)$	2
$1 \times (-5)$	-5	so	$-5 \div (-5)$	1
$0 \times (-5)$		so	$0 \div (-5)$	
$-1 \times (-5)$		so	$5 \div (-5)$	
$-2 \times (-5)$		so	$10 \div (-5)$	
$-3 \times (-5)$		so	$15 \div (-5)$	

Debriefing Questions

1. In general, when two integers with the same sign are multiplied what will be the sign of the product?
2. In general, when two integers with different signs are multiplied what will be the sign of the product?
3. In general, when two integers with the same sign are divided what will be the sign of the quotient?
4. In general, when two integers with different signs are divided what will be the sign of the quotient?





Multiplying and Dividing Integers

Elaborate

Directions: Each expression below has been simplified; however the correct simplified value may or may not be in the same box as the expression. Rearrange the simplified answers as necessary so they match their expression. After you have rearranged the values, check using your calculator. Be sure to enter the correct grouping symbols into your calculator.

What number has a prime factorization of $2^3 \times 3 \times 5$? 504	$7 - 4 \div 2 - 17$ 41	What number has a prime factorization of $2^3 \times 3^2 \times 7$? 120
$12 - 4 + 4 \times 2$ - 34	$8 + 24 \div 4 \times 3$ 0	$4 + 4(8 + 2) - 3$ 34
$4 + (-1)(2 + 6^2)$ 36	$46 - 3 \times 4 + 2$ 16	$80 - 10(2 + 3) + 4$ - 12
$9(3^2) \div (4 + 5)$ 26	$36 \div 3(5 - 2) + 6$ 9	$10 - 2(8 - 3)^2 \div 5$ 10

Debriefing Questions

1. What operation should you do first when simplifying expressions?
2. When simplifying an expression, when would you add before multiplying?
3. When would you subtract before you add?





6.3(E) The student is expected to add, subtract, multiply, and divide integers fluently.

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Question 12 - 2015

- 12** A grocery store sells steak for \$6.10 per pound. What would be the cost of $2\frac{3}{5}$ lb of steak?
- A** \$14.03
 - B** \$8.70
 - C** \$15.86
 - D** \$12.06

Answer : C

Question 2 - 2014

- 2** Mrs. Rodríguez will make name tags for each of the 45 choir members and 30 orchestra members. The materials for each name tag cost \$0.44. What is the total cost of the materials Mrs. Rodríguez will use to make these name tags?
- F** \$33.00
 - G** \$75.00
 - H** \$58.20
 - J** \$49.80

Answer : C



Question 45- 2014

- 45** One weekend Greg and Darius shoveled snow to earn spending money. Together they earned \$12.50 an hour for 9 hours of work. Each received half the total earnings. If Greg then spent \$15 on a new CD, what was the total amount he had left, in dollars and cents, from the money he earned shoveling snow that weekend?

Record your answer and fill in the bubbles on your answer document. Be sure to use the correct place value.

Answer : 41.25

Question 49- 2014

- 49** Andrew plays on a basketball team. In two games he scored $\frac{2}{5}$ of the total number of points his team scored. His team scored 55 points in the first game and 35 points in the second game. What was the number of points Andrew scored in these two games?

- A** 18
- B** 20
- C** 36
- D** 90

Answer : C



Question 15- 2013

15. In 2011 Ralph paid \$12.95 for a box of cards and \$0.44 each for 16 stamps. What was the total cost, in dollars and cents, of the box of cards and the stamps?

Record your answer and fill in the bubbles on your answer document. Be sure to use the correct place value.

Answer : 19.99

Question 16- 2013

16. Mr. Stein is purchasing 2.25 pounds of meat that costs \$2.80 per pound. How much change should Mr. Stein receive if he gives the cashier \$20.00?

- F \$6.30
- G \$13.70
- H \$14.95
- J \$2.52

Answer : G



Question 25- 2013

25. Mr. Atkinson has $5\frac{1}{4}$ lb of dry fish food. He will put an equal amount of food into 3 containers. How much fish food will be in each container?

A $1\frac{2}{3}$ lb

B $1\frac{3}{4}$ lb

C $2\frac{1}{4}$ lb

D $8\frac{3}{4}$ lb

Answer : B

Question 27- 2013

27. Wendy had 35 tickets for games at a carnival. She used $\frac{1}{5}$ of the tickets to play the ball-toss game. She then used $\frac{1}{2}$ of the remaining tickets to play the ring-toss game, in which she won 5 more tickets. How many tickets did Wendy have after playing these games?

A 7

B 19

C 14

D 28

Answer : B



Multiplying and Dividing Fractions


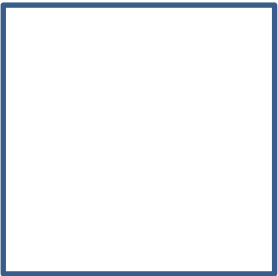


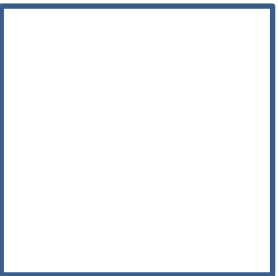

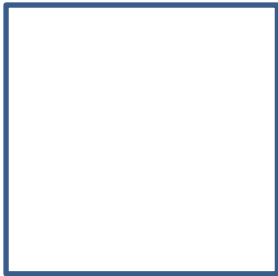
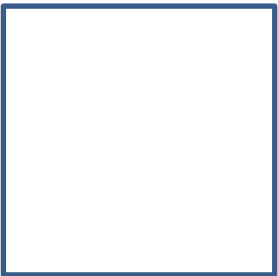

Explore

Use patty paper and complete three tasks below to obtain three in a row. You may work diagonally, horizontally, or vertically. Sketch the results of the patty paper model in the corresponding boxes on the next page.

<p>Genevieve worked on her math homework for $\frac{1}{2}$ hour last night. If she completed each problem in $\frac{1}{6}$ hour, how many math problems did she finish last night?</p> <p>_____</p>	<p>Lafayette ran $\frac{3}{4}$ of a race yesterday and walked the rest. If the length of the race was $\frac{1}{2}$ mile, how far did Lafayette run?</p> <p>_____</p>	<p>Ben ate $\frac{1}{3}$ of his share of pizza for dinner. If Ben was given $\frac{3}{4}$ of a pizza, how much did he eat?</p> <p>_____</p>
<p>Jackson rode his bicycle $\frac{2}{3}$ of a mile home. There is a stop sign every $\frac{1}{9}$ of a mile. How many stop signs are there on Jackson's ride home?</p> <p>_____</p>	<p>Janie is making cookies using a recipe that calls for $\frac{1}{3}$ cup of sugar. If she only makes $\frac{1}{2}$ of a batch, how much sugar should Janie use?</p> <p>_____</p>	<p>Veronica has $\frac{7}{8}$ yard of fabric to use to make flags. If each flag requires $\frac{1}{4}$ yard of fabric, how many can Veronica make?</p> <p>_____</p>
<p>Marcus had $\frac{5}{8}$ of the released super heroes. If he sold $\frac{1}{2}$ of his collection, what fraction of the released super heroes does Marcus have now?</p> <p>_____</p>	<p>$\frac{3}{8}$ of the students in Cassidy's class are boys. Of those students, $\frac{1}{2}$ are wearing jeans. What fraction of Cassidy's class are boys in jeans?</p> <p>_____</p>	<p>Ronnie lives $\frac{8}{9}$ mile from school. If he walked $\frac{1}{3}$ of the way home, how far did Ronnie walk?</p> <p>_____</p>



Sketch your patty paper models here:

Genevieve... 	Lafayette... 	Ben... 
Jackson... 	Janie... 	Veronica... 
Marcus... 	Cassidy... 	Ronnie... 

Debriefing Questions

1. Look at one of the multiplication models and one of the division models. What is the difference between multiplying and dividing with patty paper?
2. Explain the steps you would take to model $\frac{1}{4} \div \frac{1}{8}$ using patty paper.
3. How does the model for $\frac{1}{2} \bullet \frac{3}{4}$ relate to the solution?





7.3(B) The student is expected to apply and extend previous understandings of operations to solve problems using addition, subtraction, multiplication, and division of rational numbers.

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Question 5 - 2015

- 5 The 17 students on the math team want to raise \$483.62 to buy practice books. They have already raised \$218.25. If each student raises the same amount, how much more money must each student raise?
- A \$15.61
 - B \$12.84
 - C \$28.45
 - D \$41.29

Answer : A

Question 3 - 2014

- 3 A scientist had a bottle that contained 56.6 mL of a solution. She used 3.2 mL of the solution for an experiment. Then she poured half the solution remaining in the bottle into a beaker. Finally she poured 6 mL of the solution remaining in the bottle into a test tube. How many milliliters of solution remained in the bottle?
- A 23.7 mL
 - B 21.5 mL
 - C 19.1 mL
 - D 20.7 mL

Answer : D



Question 15- 2014

15 At a restaurant, 42 people had breakfast on Saturday morning.

- $\frac{1}{7}$ of the people ate eggs only.
- $\frac{2}{3}$ of the people ate both pancakes and eggs.
- The remaining people ate pancakes only.

Which fraction represents the number of people who ate pancakes only?

A $\frac{4}{21}$

B $\frac{8}{21}$

C $\frac{17}{21}$

D $\frac{17}{42}$

Answer : A



Question 27- 2014

27 Lindsey spent $2\frac{1}{3}$ hours in a science lab on Wednesday.

- She spent $\frac{3}{4}$ hour preparing materials for an experiment.
- She spent $\frac{5}{6}$ hour conducting the experiment.
- She spent the rest of the time cleaning her lab station.

Based on this information, which statement is true?

- A Lindsey spent $1\frac{8}{15}$ hours cleaning her lab station.
- B Lindsey spent $\frac{2}{3}$ hour preparing materials and conducting the experiment.
- C Lindsey spent the same amount of time conducting the experiment as she spent cleaning her lab station.
- D Lindsey spent $\frac{1}{12}$ hour more conducting the experiment than she spent preparing materials for the experiment.

Answer : D

Question 30- 2014

30 Roland sold candy bars for a school fund-raiser during three weeks. Some information about Roland's candy-bar sales is provided below.

- Roland sold 26 candy bars during the first week.
- The number of candy bars he sold during the second week was 4 less than 2 times the number of candy bars he sold during the first week.
- The number of candy bars he sold during the third week was 6 more than $1\frac{1}{2}$ times the number of candy bars he sold during the second week.

What was the total number of candy bars Roland sold during the three weeks?

- F 145
- G 152
- H 155
- J 142

Answer : G



Question 34- 2014

- 34** Karen had 2 spools of wire. Each spool had $15\frac{7}{8}$ yards of wire. Karen used 3 yards of wire from each spool. How many yards of wire were left on the spools?

F $41\frac{5}{8}$ yards, because $\left(15\frac{7}{8} - 2\right) \cdot 3 = 41\frac{5}{8}$

G $10\frac{7}{12}$ yards, because $\left(15\frac{7}{8} \cdot 2\right) \div 3 = 10\frac{7}{12}$

H $25\frac{3}{4}$ yards, because $\left(15\frac{7}{8} - 3\right) \cdot 2 = 25\frac{3}{4}$

J $10\frac{15}{16}$ yards, because $\left(15\frac{7}{8} \div 2\right) + 3 = 10\frac{15}{16}$

Answer : H

Question 45- 2014

- 45** One weekend Greg and Darius shoveled snow to earn spending money. Together they earned \$12.50 an hour for 9 hours of work. Each received half the total earnings. If Greg then spent \$15 on a new CD, what was the total amount he had left, in dollars and cents, from the money he earned shoveling snow that weekend?

Record your answer and fill in the bubbles on your answer document. Be sure to use the correct place value.

Answer : 41.25



Question 49- 2014

49 Andrew plays on a basketball team. In two games he scored $\frac{2}{5}$ of the total number of points his team scored. His team scored 55 points in the first game and 35 points in the second game. What was the number of points Andrew scored in these two games?

- A 18
- B 20
- C 36
- D 90

Answer : C

Question 5- 2013

5 Jackie has a puzzle book that contains a total of 65 puzzles. She has completed $\frac{2}{5}$ of the puzzles in the book. If Jackie completes 13 more puzzles, what is the total number of puzzles that she will have completed in the book?

- A 26, because $\frac{2}{5}(65) = 26$
- B 52, because $\frac{2}{5} + \frac{2}{5} = \frac{4}{5}$ and $\frac{4}{5}(65) = 52$
- C 39, because $\frac{2}{5}(65) = 26$ and $26 + 13 = 39$
- D Not here

Answer : C



Question 15- 2013

- 15** In 2011 Ralph paid \$12.95 for a box of cards and \$0.44 each for 16 stamps. What was the total cost, in dollars and cents, of the box of cards and the stamps?

Record your answer and fill in the bubbles on your answer document. Be sure to use the correct place value.

Answer : 19.99

Question 16- 2013

- 16** Mr. Stein is purchasing 2.25 pounds of meat that costs \$2.80 per pound. How much change should Mr. Stein receive if he gives the cashier \$20.00?

- F** \$6.30
- G** \$13.70
- H** \$14.95
- J** \$2.52

Answer : G

Question 17- 2013

- 17** Rachel rode her bicycle 18.2 miles in 1.4 hours. She determines that her mean speed was less than 17 miles per hour for this trip. Is she correct?

- A** No, because 1.4 added to 18.2 is 19.6
- B** No, because 18.2 multiplied by 1.4 is 25.48
- C** Yes, because 1.4 subtracted from 18.2 is 16.8
- D** Yes, because 18.2 divided by 1.4 is 13

Answer : D



Question 27- 2013

- 27 Wendy had 35 tickets for games at a carnival. She used $\frac{1}{5}$ of the tickets to play the ball-toss game. She then used $\frac{1}{2}$ of the remaining tickets to play the ring-toss game, in which she won 5 more tickets. How many tickets did Wendy have after playing these games?
- A 7
B 19
C 14
D 28

Answer : B

Question 32- 2013

- 32 The table below shows the number of cotton bales imported by the United States from different countries and regions in one year.

Cotton Imports

Country or Region	Bales (thousands)
Egypt	450
Hong Kong	475
India	1,250
Italy	1,100
Japan	925
Mexico	1,750
Taiwan	1,050

What is the mean number of cotton bales imported from these countries, in thousands?

Record your answer and fill in the bubbles on your answer document. Be sure to use the correct place value.

Answer : 1000

**Solving Problems with Rational Numbers***Explore*

Mrs. Jennings plans to bake cupcakes for the Washington Junior High Bake Sale. The recipe she plans to use is on the card below:

Recipe for: Strawberry Cupcakes makes: 1 dozen

1 package strawberry cupcake mix

$1\frac{1}{2}$ cups water

$\frac{1}{3}$ cup oil

3 eggs

1 can strawberry frosting

The Principal has determined that each parent who volunteers to contribute needs to bring 18 cupcakes. Write expressions to determine the correct amount of each ingredient Mrs. Jennings will need to use to bake 18 cupcakes.

1. Strawberry cupcake mix: _____
2. water: _____
3. oil: _____
4. eggs: _____
5. strawberry frosting: _____



Name _____ Date _____

Create a new recipe card that will yield 18 cupcakes:

Recipe for: <u>Strawberry Cupcakes</u>	makes: <u>18 cupcakes</u>
____package strawberry cupcake mix	
____cups water	
____cup oil	
____eggs	
____can strawberry frosting	

Debriefing Questions:

1. What scale factor did you use to generate the new measurements?
2. If Mrs. Jennings wanted to bring 30 cupcakes, what scale factor would she need to use to adjust the recipe?





Multiplying and Dividing Fractions

Elaborate

Part 1 Directions: Cut apart the cards below. Then, match each problem situation with the appropriate expression and solution. Paste the sets on the following page. Answer the discussion questions that follow.

<p>During a science lab, Marcus measured $\frac{1}{8}$ inch of growth on specimen A and $\frac{3}{4}$ inch on specimen B. What was the total growth of these two specimens?</p>	$\frac{3}{4} - \frac{1}{8}$	$\frac{3}{32}$
<p>6</p>	$\frac{7}{8}$	<p>Brittany's homework assignment is to read $\frac{3}{4}$ of her novel. If she has already completed $\frac{1}{8}$ of the assignment, what fraction of her novel has Brittany read?</p>
$\frac{3}{4} \times \frac{1}{8}$	<p>Aiden's track practice yesterday consisted of running $\frac{3}{4}$ mile. Today, his practice was to run $\frac{1}{8}$ mile. How much farther did Aiden run yesterday than today?</p>	$\frac{3}{4} \div \frac{1}{8}$
$\frac{5}{8}$	<p>Joshua has $\frac{3}{4}$ yard of lumber to use to make birdhouses. If each birdhouse requires $\frac{1}{8}$ yard of lumber, how many birdhouses can Joshua create?</p>	$\frac{1}{8} + \frac{3}{4}$



Debriefing Questions:

1. How did you determine which operation to use to write each expression?
2. Describe how to determine if your solution is reasonable for the number of birdhouses Joshua can create.

Part 2 Directions: Complete the table below for multiplication and division. Use the table to answer the debriefing questions.

Multiplication

Factor 1	Factor 2	Product	Product is an (<u>increase/decrease</u>) from Factor 1.
12	$\frac{1}{2}$	$12 \times \frac{1}{2} = 6$	decrease
8	$\frac{2}{3}$		
20	$\frac{7}{8}$		
$\frac{3}{5}$	$\frac{3}{4}$		
8	$\frac{5}{2}$		
6	$2\frac{1}{3}$		
$\frac{5}{8}$	$\frac{4}{3}$		
$2\frac{1}{3}$	$1\frac{3}{7}$		



Division

Division	Quotient	Multiplication	Product
$12 \div \frac{1}{2}$	$12 \div \frac{1}{2} = 24$	$12 \times \frac{2}{1}$	$12 \times \frac{2}{1} = 24$
$8 \div \frac{2}{3}$		$8 \times \frac{3}{2}$	
$\frac{3}{5} \div \frac{3}{4}$		$\frac{3}{5} \times \frac{4}{3}$	
$2\frac{1}{3} \div 1\frac{3}{7}$		$2\frac{1}{3} \times \frac{7}{10}$	

Debriefing Questions:

1. For multiplication, how does the product compare to Factor 1 when Factor 2 is less than 1? What does this tell you about how a number changes when you multiply by a number less than 1?
2. For multiplication, how does the product compare to Factor 1 when Factor 2 is greater than 1? What does this tell you about how a number changes when you multiply by a number greater than 1?
3. How does dividing by a number compare to multiplying by the reciprocal of the number?
4. Without calculation, circle the two problems below that will have the same product.

$\frac{3}{5} \div \frac{2}{7}$

$\frac{3}{5} \div \frac{7}{2}$

$\frac{5}{3} \times \frac{2}{7}$

$\frac{3}{5} \times \frac{7}{2}$

