Algebraic Reasoning Student Edition

1.3 Modeling with Linear Functions

FOCUSING QUESTION. How can you use finite differences to construct a linear model for a data set?

LEARNING OUTCOMES

- I can use finite differences to write a linear function that describes a data set.
- I can apply mathematics to problems that I see in everyday life, in society, and in the workplace.

ENGAGE

Mariette, a dendrochronologist, observed that some tree stumps have rings that are close together while other tree stumps have rings that are farther apart. Why does a tree stump have rings? What might cause the rings to be closer together or farther apart?

Image credit: Adrian Pingstone, Tree ring, Wikimedia Commons

EXPLORE

Each year during the growing season, trees grow larger by adding another layer of cells just beneath the bark. This layer is called a tree ring. Because a tree ring is added each year, scientists can determine the age of a tree by counting the number of tree rings that are present.

However, not all tree rings have the same width. Trees grow more when there is plenty of rain and the soil is fertile. Scientists can draw conclusions about temperature and rainfall for a particular year based on the width of the tree ring for that year.

Mariette measured the width of tree rings from a core sample she took from a post oak tree in the Brazos River valley of central Texas. From the tree ring width, she calculated the radius of the tree. The table below shows her results.

YEAR	2000	2001	2002	2003	2004	2005	2006
YEAR NUMBER	0	1	2	3	4	5	6
RADIUS (CM)	2.5	3.1	3.5	3.9	4.4	4.9	5.5

1. Calculate the finite differences between the year number and the radius.
2. Are the first differences in the radius constant? Explain how you know.
3. What is the average finite difference in radius?
4. Use the information from the table to write a function rule that models the data.
5. What do the slope and y-intercept from your function rule mean in the context of this situation?
6. Use your model to predict the radius of the tree in 2015.
7. In what year will the radius of the tree be 12.5 centimeters?
8. What would the circumference of the tree be in 2015?
9. Make a scatterplot of your data set and graph the function model over the scatterplot. How well would you say the function model predicts the actual values in the data set? Explain your reasoning.

REFLECT

- **How can you determine a linear function model for a data set if the first differences are not exactly the same, but are almost constant?**
- **Once you have your linear function model, how can you use the model to determine a value of the independent variable that generates a particular value of the dependent variable?**

Focusing Questions provide students with an inquiry question that will be investigated in the lesson.

Learning Outcomes revoice the state standards in student-friendly language, focusing on the particular parts of the standard that will be addressed in the lesson.

Engage activities are short mini-investigations that students can use to think about the content that is addressed in the lesson.

Explore activities use manipulatives or graphing technology to lead students through an investigation of the concept being addressed in the lesson.

Reflect prompts and questions transition students from the investigation in the Explore toward the big ideas of the lesson that will be formalized in the Explain section.

EXPLAIN

A linear function model can be used to represent sets of mathematical and real-world data. Dendrochronologists use core samples, or cylinders that are about 5 millimeters in diameter that are drilled into and extracted from the tree to measure the width of tree rings. Once they have their model, they can use different measures, such as circumference of a tree, to calculate the age of the tree.

You can use Mariette's data to generate a model relating the circumference of a tree to the age of the tree. For trees that were planted in 2000, the table shows the growth rate.

YEAR	2000	2001	2002	2003	2004	2005	2006
YEAR NUMBER	0	1	2	3	4	5	6
RADIUS (CM)	2.5	3.1	3.5	3.9	4.4	4.9	5.5

Add a new row to the table to calculate the circumference. Recall that circumference can be calculated using the formula $C = 2\pi r$, where r represents the radius of the circle and C represents the circumference of the circle. Round the circumference to the nearest tenth if necessary.

YEAR	2000	2001	2002	2003	2004	2005	2006
YEAR NUMBER	0	1	2	3	4	5	6
RADIUS (CM)	2.5	3.1	3.5	3.9	4.4	4.9	5.5
CIRCUMFERENCE (CM)	15.7	19.5	22.0	24.5	27.6	30.8	34.5

Use the rows for year number and circumference to calculate the first finite differences.

	+1	+1	+1	+1	+1	+1
YEAR NUMBER	0	1	2	3	4	5	6
CIRCUMFERENCE (CM)	15.7	19.5	22.0	24.5	27.6	30.8	34.5
	+3.8	+2.5	+2.5	+3.1	+3.2	+3.7	

1.3 • MODELING WITH LINEA

Explain sections formalize the concepts and skills presented in the lesson. Students can scan QR codes and view videos to support the Explain section.

EXAMPLE 1

A student takes small steps away from a motion detector at an approximately constant rate. The time, in seconds, for which the student walks and the distance, in meters, the student walks are recorded in the table.

TIME (S)	0	1	2	3	4
DISTANCE (M)	0.25	0.85	1.55	2.2	2.75

Generate a linear function model for this situation. Based on your model, how far away will the student be from the motion detector after 10 seconds?

STEP 1 Calculate the finite differences in the table.

		+1	+1	+1	+1
TIME (S)	0	1	2	3	4
DISTANCE (M)	0.25	0.85	1.55	2.2	2.75
		+0.6	+0.7	+0.65	+0.55

STEP 2 Calculate the average finite difference in the table and use this to determine the slope, or average velocity, for a linear function model.

$$\frac{0.6+0.7+0.65+0.55}{4} = \frac{2.5}{4} = 0.625$$

STEP 3 Use the slope and y-intercept to write a linear function model.

$y = 0.625x + 0.25$

STEP 4 Use your linear function model to make a prediction.

$y = 0.625(10) + 0.25$
$y = 6.25 + 0.25$
$y = 6.5$

According to the linear function model, the student will be 6.5 meters away from the motion detector after 10 seconds.

Examples show students how to apply the skills formalized in the Explain narrative to solve mathematical and real-world problems. Each step is stated and then clearly shown to help students understand how to create and implement a plan in a problem-solving process.

YOU TRY IT!

Tracy, a long distance runner, times herself as she runs a half-marathon, which is 13.1 miles long. The distance is measured in miles and the time is measured in minutes.

DISTANCE (MI)	1	2	3	4	5
TIME (MIN)	8.5	16.2	24.6	33.1	41.8

Generate a linear function model for this situation. Based on your model, how long to the nearest minute will it take Tracy to run the half-marathon?

You Try It! problems immediately follow each example. Students can use You Try It! problems to check their own understanding as they proceed through the lesson. Video solutions for You Try It! problems can be accessed through the QR code at the beginning of the Explain section.