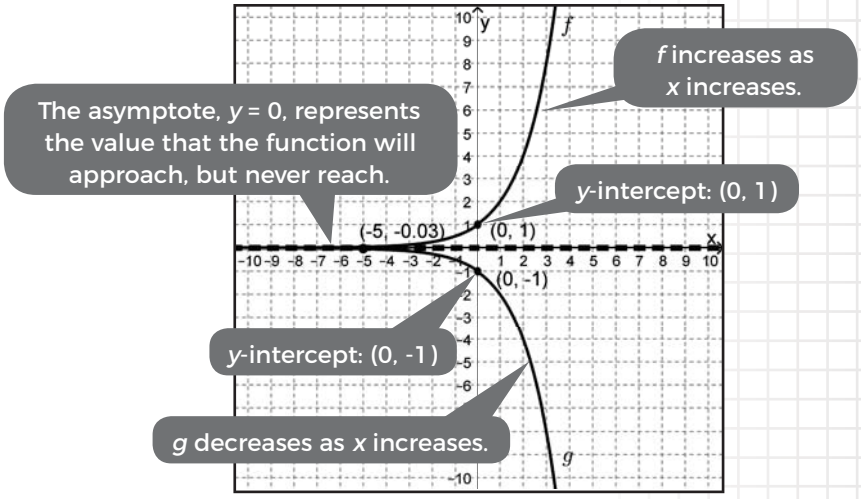
KEY FEATURES OF EXPONENTIAL FUNCTIONS

A.9D

The student is expected to graph exponential functions that model growth and decay and identify key features, including *y*-intercept and asymptote, in mathematical and real-world problems.

TELL ME MORE...

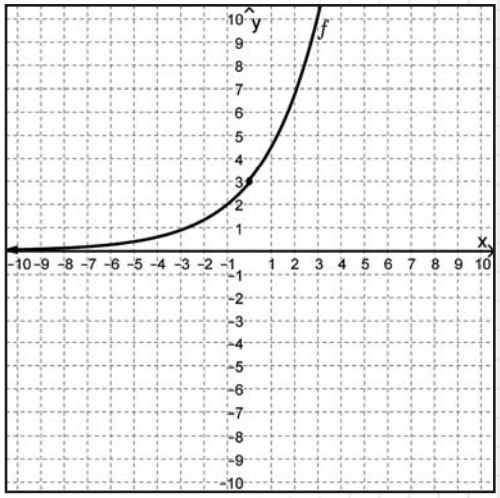
The graph of an exponential function shows certain key features that are important to the function. For example, the graphs of $f(x) = 2^x$ and $g(x) = -(2)^x$ are shown. Key features of the graphs include the *y*-intercept and horizontal asymptote.



- The ***y*-intercept** is the point where the graph of the line crosses the *y*-axis ($x = 0$).
- The **horizontal asymptote** of an exponential function is a horizontal line that represents a particular *y*-value that the exponential function will get very close to but will never be equal to. For the exponential parent function, the horizontal asymptote is the line $y = 0$, which coincides with the *x*-axis.

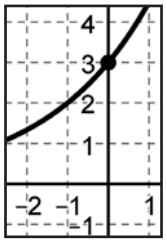
EXAMPLES

EXAMPLE 1: The graph of an exponential function, *f*, is shown. Identify the *y*-intercept and the line that best represents the asymptote of the graph.



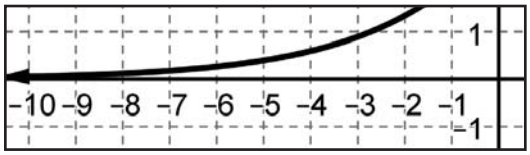
STEP 1 Identify the point where the graph of *f* crosses the *y*-axis ($x = 0$).

(0, 3)



STEP 2 Identify the asymptote as a line near a part of the graph that appears to be flat.

When $x < -7$, the graph of *f* appears to be very close to the line $y = 0$, which is the *x*-axis.



The graph of *f* is close to the line $y = 0$ when $x < -7$. The asymptote is $y = 0$.

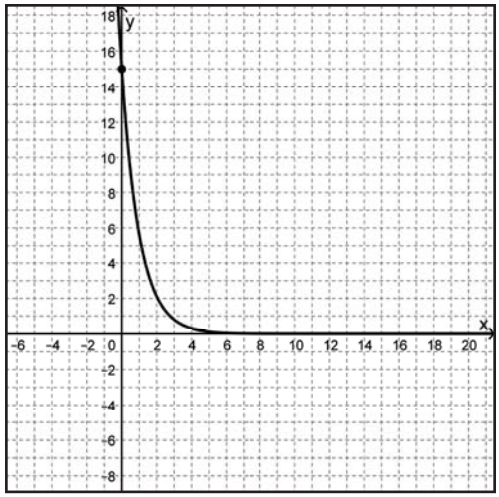
EXAMPLE 2: Which of the following statements about the graph of $y = 15\left(\frac{3}{8}\right)^x$ is true?

I. The graph has a horizontal asymptote at $y = 0$.

II. The graph has a vertical asymptote at $x = 0$.

III. The y-intercept is (0, 15).

IV. The graph decreases from left to right.



STEP 1 Graph y to get an estimate of where the asymptote and y-intercept might be.

The *y*-intercept appears to be near (0, 15). The value of *a* in the equation, 15, is the starting point. There appears to be a horizontal asymptote near the *x*-axis, *y* = 0.

STEP 2 Determine the exact location of the horizontal asymptote. Use a table of values for y around x-values that seem to line up along the asymptote.

x	4	5	6	7	8	9
y	0.297	0.111	0.042	0.016	0.006	0.002

The horizontal asymptote is *y* = 0, so Statement I is correct.

STEP 3 Look at the graph at $x = 0$ to see if there is a vertical asymptote at this point.

When $x = 0$, $y = 15$, so there cannot be an asymptote at $x = 0$.

Statement II is not correct.

STEP 4 Locate the y-intercept on the graph. Use the function to confirm its location $(0, y)$.

$$y = 15\left(\frac{3}{8}\right)^x$$

$$y = 15\left(\frac{3}{8}\right)^0$$

$$y = 15(1)$$

$$y = 15$$

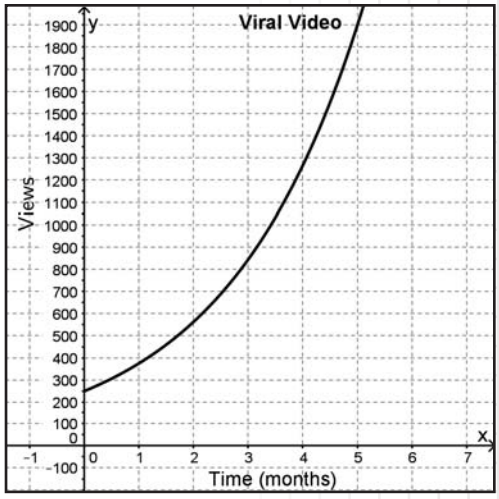
The *y*-intercept is (0, 15). Statement III is correct.

STEP 5 Determine if the function values increase or decrease as x increases from left to right.

- As x increases from left to right, function values that begin around (0, 15) quickly decrease toward $y = 0$. After $x = 5$, function values continue to slowly decrease as they gradually approach $y = 0$, the horizontal asymptote.

The graph decreases as *x* increases from left to right. Statement IV is correct.

EXAMPLE 3: The number of views of a popular online video can be modeled by the exponential function graphed on the grid, where x is the number of months since the video was posted. How many views did the video immediately receive when it was posted?



STEP 1 The independent variable, time since posting, is 0 which represents when the video was immediately posted. Determine the y-intercept.

From the graph, when $x = 0$, $y \approx 250$.

(0, 250) is the *y*-intercept.

STEP 2 Interpret the y-intercept to answer the question from the problem.

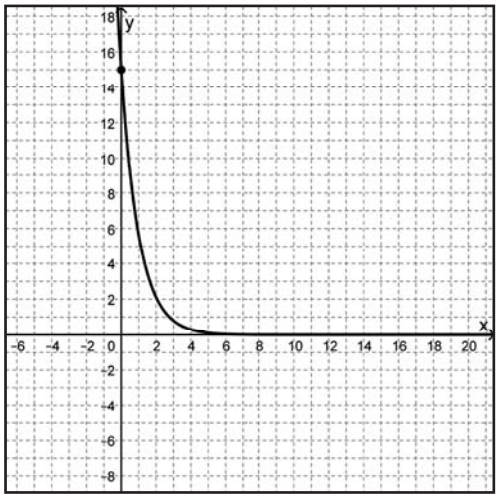
How many views did the video immediately receive when it was posted?

The video was posted at time = 0, or $x = 0$. The value of the dependent variable, the number of views, tells you the number of views when $x = 0$, or when the video was immediately posted. This number is the y-coordinate of the y-intercept.

250 views

PRACTICE

1. The graph of exponential function $k(x)$ is shown below.



What is the equation of the line that forms an asymptote for the graph of the function?

2. What are the x- and y-intercept values of $f(x) = -2(0.5)^x + 8$?

3. For the graph of exponential function $g(x) = 500(0.85)^x$, describe the behavior of the graph as it moves left to right. What is its y-intercept and the equation for any asymptote of the graph?

4. A motorcycle valued at $6,500 depreciates at a rate of 12% annually. What is the y-intercept of the graph of the function that models the situation and what is the equation of any asymptote to the function's graph?

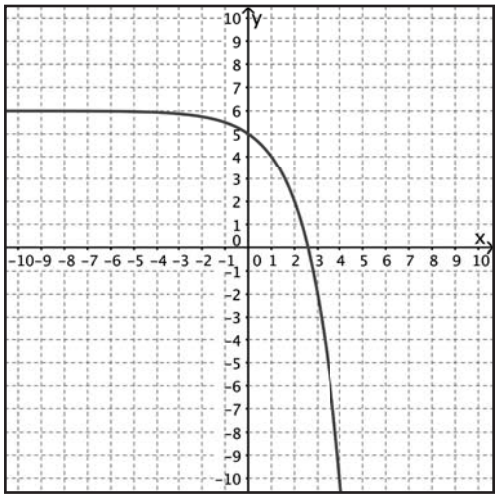
5. A biologist is studying a new species of bacteria. The biologist starts a sample of 100 bacteria and observes the growth. The function $g(x) = 100(2)^x$ models the growth of the bacteria where x represents the number of 6-hour time periods and $g(x)$ represents the number of bacteria in the culture sample. What is the shape of the graph of the function and what is the y-intercept of the graph?

6. A scientist is studying the number of mold cells present on food items based on the time it is allowed to grow and reproduce. The function $m(x) = 50(3)^x$ represents the number of mold cells present on the food for each observation time interval, x. What is the asymptote of the graph of the function? What is the y-intercept?

7. LaToya received $100 from her grandmother to open a savings account for her birthday. The account LaToya opens pays an annual interest rate of 4%. The function $y = 100(1.04)^x$ represents LaToya's bank balance, y, given the time of years, x, the account has been open. What is the y-intercept of the graph of the function? How does the graph behave as it moves from left to right?

8. The functions $f(x) = \left(\frac{1}{2}\right)^x$ and $g(x) = 2^x$ are plotted together on the same graph. What is the difference in the asymptotes of the two graphs?

9. For the exponential function shown graphed below, which is an equation for the asymptote of the function?



A $x = 4$

B $y = 6$

C $x = 6$

D $y = 4$

10. Which of the following is not true of the graph of $f(x) = \frac{5}{2}(3)^x$?

F There is an asymptote at $y = 0$.

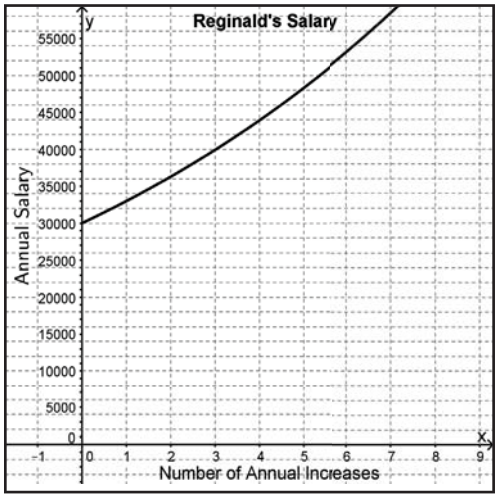
G The graph increases from left to right.

H There is an asymptote at $x = 2$.

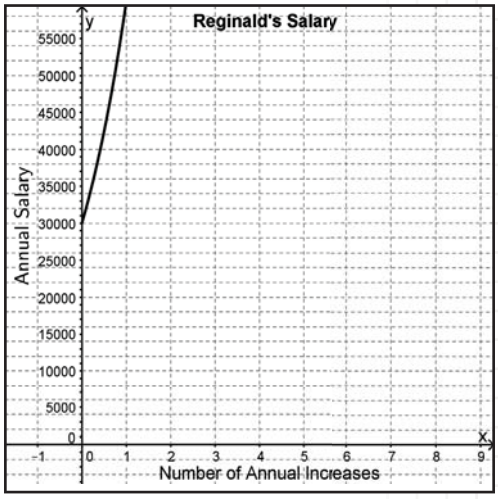
J The y-intercept is (0, 2.5).

11. Reginald received a job offer from a local company with a starting annual salary of $30,000. Each year, he will receive an annual salary increase of 10%. Which graph models this situation after Reginald receives x annual increases?

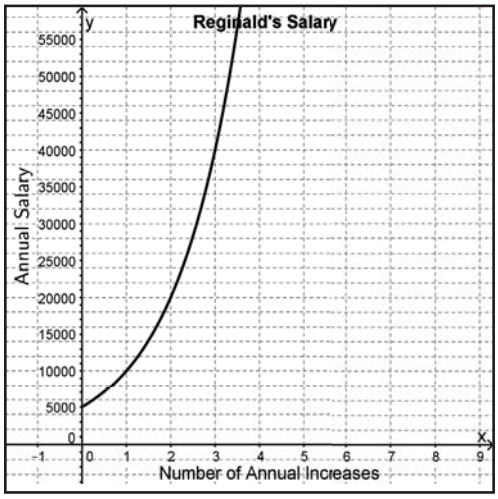
A



C



B



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