INTERPRETING EXPONENTIAL FUNCTIONS

The student is expected to interpret the meaning of the values of *a* and *b* in exponential functions of the form $f(x) = ab^x$ in real-world problems.

TELL ME MORE...

Exponential functions are based on relationships involving a constant multiplier. You can write an exponential function in general form. In this form, a represents an **initial value** or amount, and b, the **constant multiplier**, is a growth factor or factor of decay.

$$f(x) = ab^x$$

a is the initial value or starting point.

b is the growth factor or factor of decay.

Is it Growth or Decay?

The value of b answers this question.

- If $b > 1$, then the starting amount, a, is multiplied by a number greater than 1 and will increase as x increases. This is exponential growth.
- If $0 < b < 1$, then the starting amount, a, is multiplied by a number between 0 and 1 and will decrease as x increases. This is exponential decay.

EXAMPLES

EXAMPLE 1: Madison is a game warden and monitors the population of fish at a local reservoir. She uses the function $f(x) = 520(2)^x$ to determine the number of fish in the reservoir x years after the population was stocked in 2010. What do the numbers 520 and 2 mean in the population model?

STEP 1 Determine the values of a and b in the function $f(x)$. The coefficient is the value of a and the exponential base is the value of b.

$f(x) = 520(2)^x$

$a = 520$ $b = 2$

$a = 520$ and $b = 2$

STEP 2 Interpret the value of a as a starting point or initial value.

- The problem states that $f(x)$ represents the population, or number of fish in the reservoir.
- The problem states that the population of fish was stocked in 2010, meaning that's when the game warden placed fish into the reservoir.

The reservoir was stocked with 520 fish in the year 2010.

STEP 3 Interpret the value of b as the growth rate or rate of decay.

- $b = 2$, which is greater than 1, so this value is a growth rate.
- Multiplying by a factor of 2 results in each year being double the previous year.

The fish population doubles, or increases by a factor of 2, each year.

EXAMPLE 2: Marquise purchased a new motorcycle. The motorcycle depreciates, or loses value, each year. The table contains values of the exponential function $v(x)$. In this function, x represents the number of years since Marquise purchased the motorcycle and $v(x)$ represents the value of the motorcycle. If $v(x) = ab^x$, what do the values of a and b mean in the context of this situation?

Time (years), x	Value (dollars), $v(x)$
0	9000
1	8325
2	7700.63
3	7123.08
4	6588.85

STEP 1 Determine the constant multiplier, or constant ratio, between successive function values. This multiplier will be the value of b.

Time (years), x	Value (dollars), $v(x)$	
0	9000	
1	8325	$8325 \div 9000 = 0.925$
2	7700.63	$7700.63 \div 8325 \approx 0.925$
3	7123.08	$7123.08 \div 7700.63 \approx 0.925$
4	6588.85	$6588.85 \div 7123.08 \approx 0.925$

***b* = 0.925**

MAKE A NOTE . . .

What happens when you multiply a number by 100%, a number less than 100%, or a number greater than 100%? Record your thoughts here.

STEP 2 Interpret the value of b as the growth rate or rate of decay.

- $b = 0.925$, which is less than 1, so this value is a rate of decay and the function values will decrease as x increases.
- 0.925 as a percent is 92.5%. Subtract this value from 100% to determine the annual rate of depreciation.

The value of the motorcycle decreases by 7.5% each year.

STEP 3 Identify the initial value, or the function value when $x = 0$. This value will be the value of a.

***a* = 9000**

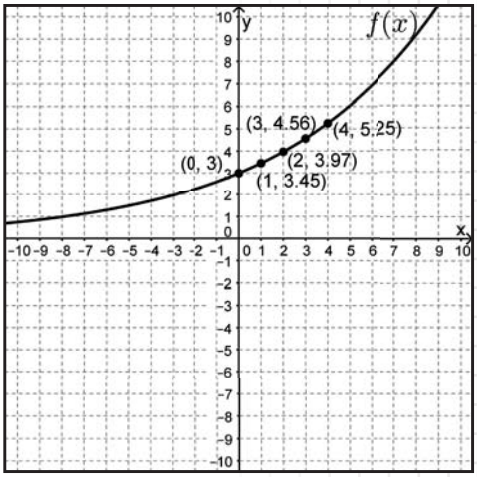
Time (years), x	Value (dollars), $v(x)$
0	9000
1	8325
2	7700.63
3	7123.08
4	6588.85

STEP 4 Interpret the value of a as a starting point or initial value.

- The problem states that $v(x)$ represents the value of the motorcycle.
- Year 0 was when Marquise purchased the motorcycle.

Marquise purchased the motorcycle for $9000.

EXAMPLE 3: The graph of $f(x)$, an exponential function, is shown. The function $f(x)$ represents the population, in thousands, of a town in West Texas, after oil was discovered in 2010. Let x represent the number of years since 2010. If $f(x)$ is of the form $f(x) = ab^x$, what do the values of a and b mean?



STEP 1 Determine the constant multiplier, or constant ratio, between successive function values. This multiplier will be the value of b. Use any two consecutive values of $f(x)$ from the graph.

$$b = 3.45 \div 3 = 1.15$$

***b* = 1.15**

STEP 2 Interpret the value of b as the growth rate or rate of decay.

- $b = 1.15$, which is greater than 1, so this value is a growth rate.
- 1.15 as a percent is 115%. Subtract 100% from this value to determine the annual rate of increase.

The population increases by 15% each year.

STEP 3 Identify the initial value, or the function value when $x = 0$. In a graph, this value is the y-intercept and the y-coordinate of the y-intercept will be the value of a.

The y-intercept is (0, 3), so $a = 3$.

***a* = 3**

STEP 4 Interpret the value of a as a starting point or initial value.

- The problem states that $f(x)$ represents the population x years after oil was discovered in 2010.
- Year 0 was when oil was discovered.

The population in 2010 was 3,000.

PRACTICE

Use the following information for Questions 1 and 2.

A bear is given a tranquilizer in order for veterinarians to provide care to the animal. The amount of milligrams of medication remaining in the bear's blood stream based on the number of hours since the injection is modeled with the function $f(x) = 250(0.75)^x$.

1. What is the meaning of the value 250 in the function model?

2. What is the meaning of the value 0.75 in the function model?

Use the following information for Questions 3 and 4.

Bacteria is growing exponentially in a laboratory as shown in the table below. The researcher observes the number of bacteria in the sample each hour after starting the sample. She writes a function to model the bacteria's growth in the form of $f(x) = ab^x$, where x represents the time since starting the sample.

Hours, x	0	1	2	3	4	5
Population, $f(x)$	15	30	60	120	240	480

3. What is the value and meaning of the variable a in the function modeling the bacteria's growth?

4. What is the value and meaning of the variable b in the function modeling the bacteria's growth?

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5. A rodent species has a population of 25,000 and is decreasing in size at a rate of 15% per year. When the function is written to model the situation, what are the values of a and b used in the function model?

6. The function $f(x) = 300(1.06)^x$ models the value of Mario's investment in a stock fund as a function of time in years. Which of the following is true?

A Mario initially invested \$300 and his money increases by 1.06 times the number of years invested.

B Mario initially invested \$300 and his money decreases by1.06 times the number of years invested.

C Mario initially invested \$300 and his money increases by 6% each year invested.

D Mario initially invested \$300 and his money decreases by 6% each year invested.

7. The table below shows values for an exponential function, $N(x)$.

Time (years), x	**Number in sample, $N(x)$**
0	2
1	24
2	288
3	3,456
4	41,472

Which of the following best describes the situation represented by the function?

F The number of weeds increases by a factor of 12 each year compared to the year before.

G There were 12 weed plants in the sample when the growth started.

H The number of weeds increases by 12 times the number of years growing.

J The number of weeds in the sample is decreasing by a factor of 12 each year compared to the year before.