

# DOMAIN AND RANGE OF EXPONENTIAL FUNCTIONS



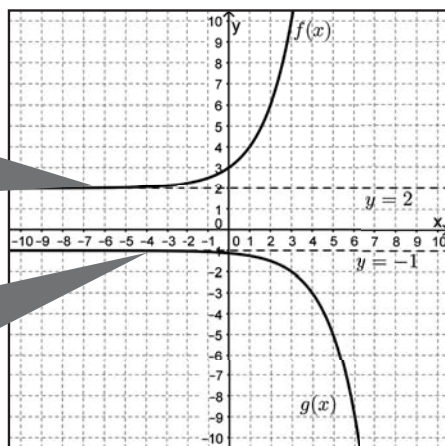
The student is expected to determine the domain and range of exponential functions of the form  $f(x) = ab^x$  and represent the domain and range using inequalities.

## i TELL ME MORE...

The **domain** of a function is the set of input values that are used for the independent variable. The **range** of a function is the set of output values for the dependent variable. For any exponential function,  $f(x) = ab^x$ , the domain is the set of all real numbers. The range, however, is bounded by the horizontal asymptote of the graph of  $f(x)$ . Use the graph to identify the range of  $f(x)$  and  $g(x)$ .

$f(x) = 2^x + 2$   
**Asymptote:  $y = 2$ ,  $a > 0$**   
 Range does not include the asymptote. All real numbers greater than \_\_\_\_\_.

$g(x) = -(2)^{x-3} - 1$   
**Asymptote:  $y = -1$ ,  $a < 0$**   
 Range does not include the asymptote. All real numbers less than \_\_\_\_\_.



### Domain

For any exponential function,  $f(x) = ab^x$ , the domain is the set of all real numbers.

$$-\infty < x < \infty$$

### Range

For any exponential function,  $f(x) = ab^x$ , the range is the set of real numbers above or below the horizontal asymptote,  $y = d$ , but does not include  $d$ , the value of the asymptote.

$$\begin{aligned} \text{If } a > 0: f(x) &> d \\ \text{If } a < 0: f(x) &< d \end{aligned}$$

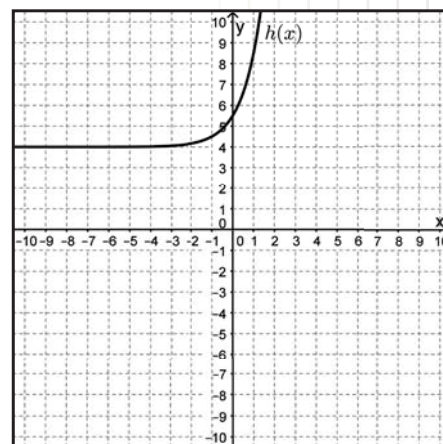
However, when an exponential function is used to model real-world situations, then the domain and range can be restricted to numbers that make sense for the situation. In these cases, the domain or range can be written using inequalities for a **continuous** interval.

## ✓ EXAMPLES

**EXAMPLE 1:** What are the domain and range of  $h(x) = 1.5(3)^x + 4$ ?

- STEP 1** Graph the function to visually inspect the possible range.
- Function values increase as  $x$  increases.
  - There appears to be a horizontal asymptote near  $y = 4$ .
  - The value of  $a$  is positive.

**The possible range is  $h(x) > 4$ .**



**STEP 2** Use the parameters in the equation to confirm the location of the asymptote.

- In  $h(x)$ ,  $d = 4$ . Thus, the horizontal asymptote is  $y = 4$ .
- In  $h(x)$ ,  $a = 1.5$  which is greater than 0.

**The horizontal asymptote is  $y = 4$ .**

**STEP 3** State the domain and range of the function.

**The domain of  $h(x)$  is all real numbers.**

**The range of  $h(x)$  is all real numbers greater than 4, or  $h(x) > 4$ .**

**EXAMPLE 2:** Charnette purchased a new vehicle. Vehicles depreciate, or lose value over time. Charnette's vehicle depreciates according to the function  $f$  whose graph is shown. What is the domain and range of  $f$ ?

**STEP 1** Determine any limits on the independent variable, time, or dependent variable, value.

- In this situation, time is non-negative. It can be 0 or any real number greater than 0, since time is given in years since Charnette purchased the vehicle.
- In this situation, the value of Charnette's car when it was brand new (time = 0) appears to be \$18,500. Thus, it cannot exceed this value. The value will also never be less than \$0.

**Both variables are limited to non-negative numbers (y-axis and Quadrant I) in this situation.**

**STEP 2** State the domain of  $f$  for this situation, keeping in mind the constraints already observed.

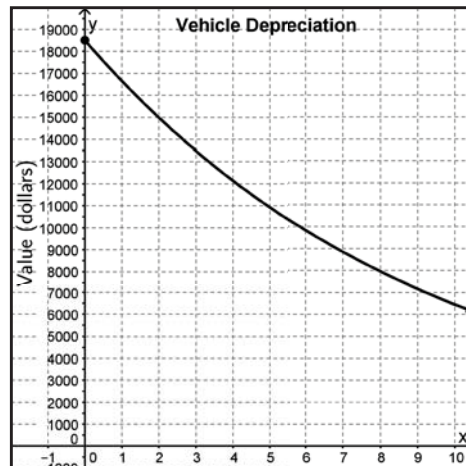
- $x$  can be any real number greater than or equal to 0.

**The domain of the function in this situation is  $x \geq 0$ .**

**STEP 3** State the range of the function for this situation, keeping in mind the constraints already observed.

- $y$ , which represents the function values for  $f(x)$ , can be any real number greater than 0 and less than or including 18,500.

**The range of the function in this situation is  $0 < y \leq 18,500$ .**



**YOU TRY IT!**

The table contains some points on the graph of an exponential function. What is the range of the function?

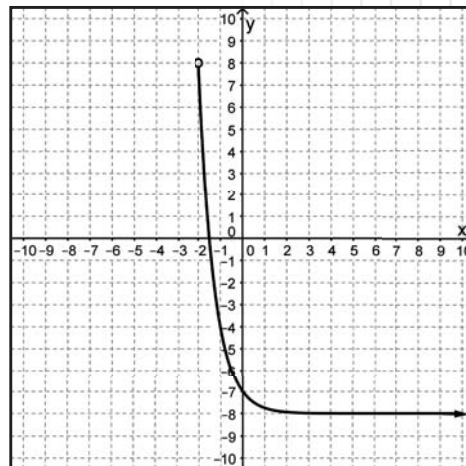
$x$	$y$
0	6
1	3
2	1.5
3	0.75
4	0.375

Value of  $a$ : \_\_\_\_\_  $b$ : \_\_\_\_\_

Function: \_\_\_\_\_

Range: \_\_\_\_\_

**EXAMPLE 3:** The graph of a part of exponential function  $m$  is shown. Write the domain and range of the part shown using inequalities.



**STEP 1** Locate any vertical or horizontal limits such as endpoints or asymptotes.

- open endpoint at  $(-2, 8)$
- horizontal asymptote at  $y = -8$

**STEP 2** Determine any restrictions from the open endpoint.

- There are no  $x$ -values to the left of the open endpoint. The lower limit of the domain is  $-2$ . Since the endpoint is open,  $-2$  is not included in the domain.
- There are no function values above the endpoint, so the upper limit of the range is  $8$ . Since the endpoint is open,  $8$  is not included in the range.

**$-2$ , not inclusive, is the lower limit of the domain.  
 $8$ , not inclusive, is the upper limit of the range.**

**STEP 3** Determine how the asymptote affects the domain and range.

- The horizontal asymptote is the lower limit of the range since the function values will approach, but never equal, the  $y$ -value of the asymptote.
- The domain continues to the right without limit.

**$-8$ , not inclusive, is the lower limit of the range.**

**STEP 4** Write the domain using the lower limit, inequalities, and the upper limit. If the limit is included in the domain, be sure the inequality contains “or equal to.”

$$-2 < x$$

**STEP 5** Write the range using the lower limit, inequalities, and the upper limit. If the limit is included in the range, be sure the inequality contains “or equal to.”

$$-8 < y < 8$$



## PRACTICE

For questions 1-5 identify the domain and range for each function using inequalities as appropriate.

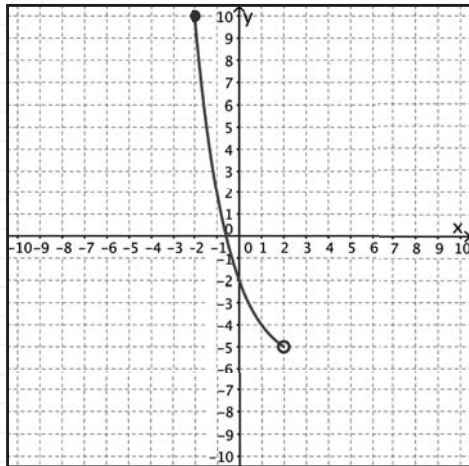
1.  $f(x) = \frac{1}{2}(3)^x + 2$  when  $x \leq 4$

3. The table below shows points that belong to the function  $g(x)$ .

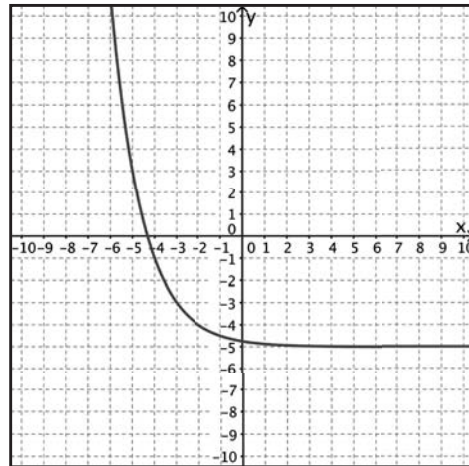
$x$	-1	0	1	2
$g(x)$	$\frac{1}{3}$	1	3	9

2.  $b(x) = 30\left(\frac{1}{2}\right)^x$

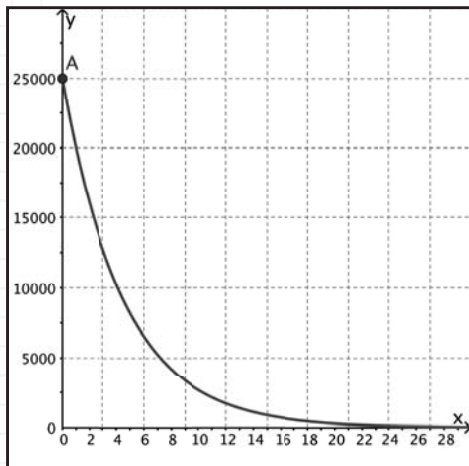
4.



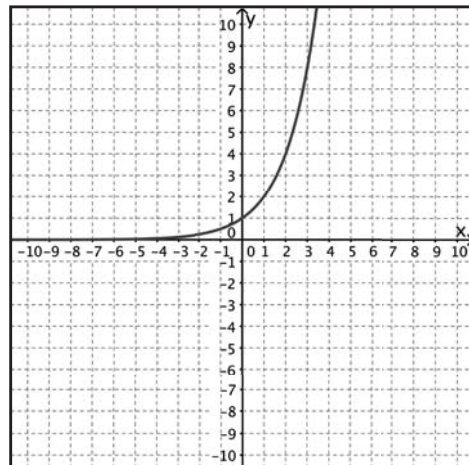
5.



6. The graph below shows a function representing the population of mice,  $y$ , in a growing town over a number of years,  $x$ . What inequality represents the range of the population of mice in the situation?



9. The graph of an exponential function is shown below.



7. The volume,  $V$ , of air remaining in an inflated balloon can be modeled by the function  $V = 1,000(0.85)^x$  where  $x$  represents the number of days that have passed since inflating the balloon. What is the reasonable domain for the situation?
8. The function  $f(p) = 65,000(1.05)^x$  can be used to model the population of a city for  $x$ , the number years that have passed since 2000. What inequality represents the reasonable range of the function based on the situation?

Which of the following statements is true about the function?

- A The range of the function is the set of all real numbers.
- B The domain of the function is the set of all real numbers less than 4.
- C The range of the function is the set of all real numbers less than 4.
- D The domain of the function is the set of all real numbers.